

MODELING A POPULATION'S HEALTH PREFERENCES: A BAYESIAN APPROACH

Refik Soyer
Department of Decision Sciences
The George Washington University

Joint work with

Muzaffer Musal
Texas State University-San Marcos

Christopher McCabe
University of Leeds

Samer A. Kharroubi
University of York

OUTLINE OF THE TALK

- Introduction and Motivation
 - Preference based measurement of health
 - Health related quality of life measures
- Utility of health states
 - Composite vs decomposed methods
 - Health Utility Index 2
- Bayesian Framework
 - modeling population utilities
 - modeling multiattribute utility function
- Estimating utilities of health states
- Illustration with actual data

PREFERENCE BASED MEASUREMENT OF HEALTH

Preference based measurement of health (PBMH) have been developed to be used in economic evaluation of health policies.

Use of preference based measures requires quantification of health state preferences by a group of individuals.

This preference data is used as a sample to develop an aggregate measure for the population.

The methods that quantify preference based measurement of health (PBMH) are referred to as the health related quality of life measures (HRQoL).

These measures are used to quantify a population's preferences over health states as well as of a treatment's effect.

- Health Utilities Index (HUI): Torrance et. al (1995)
- Quality of Well-Being (QWB) scale: Kaplan et. al (1988)
- Short Form (SF-6D) survey: Brazier, Roberts and Deverill (2002).

PBMH AND UTILITY

These measures are based on a multi attribute model for evaluating health states using preference weights and scores.

They provide a single index number for each health state. Typically an index value "1" denotes perfect health and "0" denotes death.

In the health economics literature, these index values are referred to as *utility*.

Health states have multiple dimensions allowing a multi attribute model.

Elicitation of utility requires sophisticated procedures based on standard gambles as discussed; Brazier and Deverill (1999).

A more simplified approach for obtaining preference measures is to ask respondents to assign values to health states directly and have the analyst convert these to utility.

COMPOSITE METHODS

Preference based measures such as QWB and SF-6D use what is referred to as the *composite* approach for estimation of the multi attribute utility function for the health states.

The composite approach involves direct elicitation of utility of multidimensional health states and the approach requires more health states than that can be evaluated by a single respondent [Brazier (2005)].

Regression models are used to extrapolate the values of health states that are not included in the survey.

An alternative for estimation of the utilities for the health states is the *decomposed* approach employed by the HUI.

This method uses the MAUT framework developed by Keeney and Raiffa (1976) and determines a functional form for the multi attribute utility function of health states.

DECOMPOSED APPROACH

- Based on simplifying assumptions:

- *preferential independence* and *utility independence*

The approach yields simpler forms of utility functions and substantially reduces the valuation effort by decomposing the problem into one-dimensional elicitation problems.

Hazen (2004) describes how the additive or multiplicative decomposition within QALYs can be constructed using these independence concepts and discusses how they relate to HUI.

In addition to providing evaluations of all possible health states, the decomposed approach is also flexible in modeling interactions using multiplicative utility functions of Keeney (1974).

This is unlike the composite approach where there is no standard method for determining the states required to estimate a model with interaction terms; Brazier (2005).

PBMH DATA

Both the composite and decomposed approaches provide us with a sample of health state valuation data, that is, with health state utilities from a sample of individuals.

The objective is to estimate the health state utilities of the population based on this sample and use the estimated population utility function to evaluate different health policy alternatives.

Statistical methods have been considered by earlier researchers such as Dolan (1997) and Brazier et al. (2002). In general these approaches employed linear models with normally distributed error terms.

Brazier et al. (2002) proposed extensions to include random effect terms.

As pointed out by Brazier (2005), these models, that used data from the composite approach, "have estimated crude summary terms for interactions" and have required range of transformations to deal with highly skewed data.

BAYESIAN WORK

- A nonparametric Bayesian approach for estimation of the HRQoL of a population.
 - Kharroubi, O'Hagan and Brazier (2005)
 - Kharroubi et. al (2007)

Utility quantifies the HRQoL measured by the SF-6D which is based on six attributes each having 4 to 6 levels \Rightarrow 18,000 possible health states.

Use of standard gamble techniques to elicit utilities of health states from the sample of individuals (a sampling method where an individual goes through the standard gambles for a limited number of health states).

Utility function is estimated from these health states using a multivariate nonparametric Bayesian model.

The approach is based on the composite approach that requires large number of evaluations of health states.

OUR APPROACH

- Although the composite approach involves cognitively complex tasks in elicitation, most of the literature on statistical modeling of health state data is limited to this approach; Brazier (2005)
- We consider parametric Bayesian models and make inferences to analyze health state utility data using a decomposed approach.

We describe uncertainty about the unknown utility function probabilistically and pose the population utility estimation problem as a Bayesian inference problem.

Our framework is different than Kharroubi, O'Hagan and Brazier (2005) due to its parametric nature and its use of a decomposed approach based on a multi attribute utility function

- modeling attribute utilities
- modeling attribute weights
- using a multiattribute model for aggregation

Our approach uses HUI2 to describe the population's HRQoL.

PBMH METHODS

	Dimension			Valuation Characteristics	
	Dimension	Levels	Health States	Valuation technique	Method of Extrapolation
QWB	Mobility, physical activity, social functioning 27 symptoms/problems	3-2	1,170	VAS	Statistical
HUI-2	Sensory, mobility, emotion, cognitive, self-care, pain, fertility	3-5	2,400	VAS transformed into SG	MAUT
HUI-3	Vision, hearing, speech, ambulation, dexterity, emotion, cognition, pain	5-6	972,000	VAS transformed into SG	MAUT
EQ-5D	Mobility, self-care, usual activities, pain/discomfort, anxiety/depression	3	243	TTO and VAS	Statistical
SF-6D	Physical functioning, role limitation, social functioning, pain, energy, mental health	4-6	18,000	SG	Statistical
Excerpt From Table1- Brazier, "Current State of the Art in Preference-Based Measures of Health and Avenues for Further Research" Discussion Paper 2004					

BAYESIAN MODELING OF UTILITIES

The HUI measurement problem is a multi attribute evaluation problem with C attributes where a health state is defined as the composite description of C attributes' levels.

The objective is to estimate the utility of a population's health state based on the utilities obtained from a sample of N individuals

⇒ need a probability model for describing population utility.

Following the decomposed approach of Torrance et al. (1996), we will use multiplicative utility functions ⇒ mutual utility independence of the attributes.

This allows us to model population utility for each of the attributes and estimate population utility on any given dimension.

Once estimation is completed for all attributes independently, population's multi attribute utility (MAU) function can be obtained via the multiplicative utility model.

MODELING POPULATION UTILITIES

Assume that preference ordering of the $K_c + 1$ levels with respect to each attribute $c = 1, \dots, C$ is identical for the population

$$X_{c,1} \prec X_{c,2} \prec \dots \prec X_{c,K_c} \prec X_{c,K_c+1}$$

$X_{c,j} \equiv j$ th level of a single attribute c .

We are interested in making inference about the unknown population utilities

$$u(X_{c,2}) < \dots < u(X_{c,K_c}),$$

where $u(X_{c,1}) = 0$ and $u(X_{c,K_c+1}) = 1$.

We may have a prior opinion on these values and we are interested in updating this prior opinion based on the sample utility measurements on the N individuals.

In general $u(X_{c,j}^i) = u_{c,j}^i \equiv$ utility declared by the i -th individual for attribute c at level j .

A MODEL FOR POPULATION UTILITIES

- We focus on a single criterion and to reflect the ordering that applies to the population we assume that for all individuals

$$0 < u_2 < u_3 < \cdots < u_K < 1.$$

We want a probability model for the utility vectors $\mathbf{u}^i = (u_1^i, u_2^i, \dots, u_K^i, u_{K+1}^i)$ which is consistent with the ordering and flexible enough to reflect the diminishing utility scenario encountered in many applications.

- Ordered Dirichlet distribution: Mazzuchi and Soyer (1993)

The ordered Dirichlet model for $\mathbf{u} = (u_2, u_3, \dots, u_K)$ is given by

$$p(\mathbf{u} \mid \beta, \boldsymbol{\alpha}) = \frac{\Gamma(\beta)}{\prod_{j=2}^{K+1} \Gamma(\beta\alpha_j)} \prod_{j=2}^{K+1} (u_j - u_{j-1})^{\beta\alpha_j - 1},$$

where $u_1 = 0$ and $u_{K+1} = 1$.

PROPERTIES OF THE MODEL

- β and $\boldsymbol{\alpha} = (\alpha_2, \alpha_3, \dots, \alpha_{K+1})$ are the parameters such that $\alpha_j > 0$ and $\sum_{j=2}^{K+1} \alpha_j = 1$.
- Distribution is defined over the simplex $\left\{ \mathbf{u} \mid 0 < u_2 < u_3 < \dots < u_K < 1 \right\}$
- The model implies that changes in the utilities $(u_j - u_{j-1})$, for $j = 2, \dots, K + 1$, follow a Dirichlet distribution.

- Marginals are: $(u_j \mid \beta, \boldsymbol{\alpha}) \sim \text{Beta}\left(\beta\alpha_j^*, \beta(1 - \alpha_j^*)\right)$

for $j = 2, \dots, K$, where $\alpha_j^* = \sum_{k=2}^j \alpha_k \Rightarrow E[u_j \mid \beta, \boldsymbol{\alpha}] = \alpha_j^*$.

- Marginal utilities

$$(u_j - u_i) \mid \beta, \boldsymbol{\alpha} \sim \text{Beta}\left(\beta(\alpha_j^* - \alpha_i^*), \beta(1 - \alpha_j^* + \alpha_i^*)\right) \text{ for } i < j,$$

For adjacent levels

$$E[u_j - u_{j-1} \mid \beta, \boldsymbol{\alpha}] = (\alpha_j^* - \alpha_{j-1}^*) = \alpha_j.$$

UTILITY INTERPRETATIONS

- α_j can be interpreted as the expected increase in utility as a result of going from attribute level X_{j-1} to attribute level X_j .
- α_j^* is the expected utility at attribute level X_j .
- α_j^* is increasing with j , implying that for the population we expect utility is an increasing function of the attribute when high values of the attribute are desirable.
- If $E[u_j - u_{j-1} \mid \beta, \alpha] = \alpha_j$ is a decreasing sequence in j , then we expect that the marginal utility is diminishing as the attribute level gets larger.

$$\Rightarrow E[u_j \mid \beta, \alpha] = \alpha_j^* \text{ is discrete concave in } j.$$

- Note that we may have prior beliefs about the general behavior of the expected utility function and we can incorporate that in our Bayesian analysis. Such prior beliefs can be used in specification of the prior distribution of the parameters α and β .

BAYESIAN INFERENCE FOR POPULATION UTILITIES

We are interested in describing uncertainty about the population utility

$\mathbf{u} = (u_2, \dots, u_K)$ based on the information provided by the sample of N utility vectors $\mathbf{u}^i = (u_2^i, u_3^i, \dots, u_K^i)$, $i = 1, \dots, N$.

- Given sample utilities $\mathbf{u}^{(N)} = (\mathbf{u}^1, \mathbf{u}^2, \dots, \mathbf{u}^N)$ from the N individuals, we update our knowledge about \mathbf{u} via the calculus of probability

\Rightarrow posterior predictive distribution $p(\mathbf{u}|\mathbf{u}^1, \mathbf{u}^2, \dots, \mathbf{u}^N)$.

- Once the prior $p(\beta, \alpha|\mathcal{H})$ is specified, we can revise our uncertainty via Bayes' rule

$$p(\beta, \alpha|\mathbf{u}^{(N)}) \propto L(\beta, \alpha; \mathbf{u}^{(N)}) p(\beta, \alpha|\mathcal{H})$$

where $L(\beta, \alpha; \mathbf{u}^{(N)})$ is based on the ordered Dirichlet distribution.

- Computation of $p(\beta, \alpha|\mathbf{u}^{(N)})$ is via MCMC.

POSTERIOR PREDICTIVE DISTRIBUTION

Once the posterior density $p(\beta, \alpha | \mathbf{u}^{(N)})$ is evaluated, we need to evaluate

$$p(\mathbf{u} | \mathbf{u}^{(N)}) = \int_{\beta, \alpha} p(\mathbf{u} | \beta, \alpha, \mathbf{u}^{(N)}) p(\beta, \alpha | \mathbf{u}^{(N)}) d\alpha d\beta,$$

$$p(\mathbf{u} | \mathbf{u}^{(N)}) = \int_{\beta, \alpha} p(\mathbf{u} | \beta, \alpha) p(\beta, \alpha | \mathbf{u}^{(N)}) d\alpha d\beta.$$

- Given the posterior samples $(\beta^{(s)}, \alpha^{(s)})_{s=1}^S$ from $p(\beta, \alpha | \mathbf{u}^{(N)})$, we can approximate $p(\mathbf{u} | \mathbf{u}^{(N)})$ as

$$p(\mathbf{u} | \mathbf{u}^{(N)}) \approx \frac{1}{S} \sum_{s=1}^S p(\mathbf{u} | \beta^{(s)}, \alpha^{(s)}).$$

- It can be extended to C attributes and the posterior utility distributions $p(\mathbf{u}_c | \mathbf{u}_c^{(N)})$ are obtained for $c = 1, \dots, C$ independently.

MODELING MULTIATTRIBUTE UTILITY FUNCTION

- A common method of decomposition is the multiplicative utility model

$$1 + ku(X_{1,A_i}, \dots, X_{C,A_i}) = \prod_{c=1}^C (1 + k k_c u(X_{A_i})),$$

where $0 < k_c < 1$ and $k > -1$ are scaling coefficients; $k = 0 \Rightarrow$ additive model.

- Assume that the k_c 's are independently beta distributed

$$(k_c \mid \kappa, \nu_c) \sim \text{Beta}(\kappa \nu_c, \kappa(1 - \nu_c)),$$

where $0 < \nu_c < 1$ is the expected value of k_c , for $c = 1, \dots, C$, and κ represents the strength of belief parameter.

- Given the prior $p(\boldsymbol{\nu}, \kappa \mid \mathcal{H})$, where $\boldsymbol{\nu} = (\nu_1, \dots, \nu_C)$ and the sample of coefficients $\mathbf{K}^N = (\mathbf{k}^i = (k_1^i, \dots, k_C^i); i = 1, \dots, N)$ elicited from N individuals, we can obtain

$$p(\boldsymbol{\nu}, \kappa \mid \mathbf{K}^N) \propto L(\kappa, \boldsymbol{\nu}; \mathbf{K}^N) p(\boldsymbol{\nu}, \kappa \mid \mathcal{H}),$$

where $L(\kappa, \boldsymbol{\nu}; \mathbf{K}^N)$ is a product of beta densities.

ESTIMATION OF THE MULTIATTRIBUTE UTILITY FUNCTION

- Once $p(\kappa, \boldsymbol{\nu} | \mathbf{K}^N)$ is obtained, we can evaluate the posterior predictive distribution

$$p(\mathbf{K} | \mathbf{K}^N) = \int_{\kappa, \boldsymbol{\nu}} p(\mathbf{K} | \kappa, \boldsymbol{\nu}) p(\kappa, \boldsymbol{\nu} | \mathbf{K}^N) d\kappa d\boldsymbol{\nu},$$

which can be approximated by using S realizations from $p(\kappa, \boldsymbol{\nu} | \mathbf{K}^N)$ as

$$p(\mathbf{K} | \mathbf{K}^N) \approx \frac{1}{S} \sum_{s=1}^S p(\mathbf{K} | (\kappa, \boldsymbol{\nu})^{(s)}).$$

- Given the above the posterior predictive distribution of k can be obtained via simulation using the identity

$$1 + k = \prod_{c=1}^C (1 + k k_c).$$

For each realization, $\mathbf{K}^{(s)}$, $s = 1, \dots, S$, the above identity can be solved for the interaction parameter k and its posterior predictive distribution can be obtained.

ESTIMATING UTILITIES OF HEALTH STATES

- Given samples from the posterior predictive distributions

$$p(\mathbf{u}_c | \mathbf{u}_c^{(N)}), c = 1, \dots, C \text{ and } p(\mathbf{K} | \mathbf{K}^N)$$

we can evaluate the population utility distribution for a specific health state, say A_i .

This is done by using a Monte Carlo evaluation of $u(X_{1,A_i}, \dots, X_{C,A_i})$ using the multiplicative utility model.

- We can make probability statements on whether health state A_i is preferred to A_j , $Pr\{A_i \succ A_j | \mathbf{K}^N, \mathbf{u}_1^{(N)}, \dots, \mathbf{u}_C^{(N)}\}$ which is equivalent to

$$Pr\left\{u(X_{1,A_i}, X_{2,A_i}, \dots, X_{C,A_i}) > u(X_{1,A_j}, X_{2,A_j}, \dots, X_{C,A_j}) \mid \mathbf{K}^N, \mathbf{u}_1^{(N)}, \dots, \mathbf{u}_C^{(N)}\right\}$$

which can be approximated using the MC samples

- Thus, we can compute the posterior probability that a particular health state is preferred to another for a randomly selected individual from the population.

EXTENSIONS OF THE MODEL

- Incorporating covariate effects (such as age, gender, current health status, etc.)
 - effects of covariates on utility at a given attribute level.
 - effects of covariates on scaling coefficients, k_c 's, of the multiplicative model.

We can model the expected changes in utility between the adjacent levels of an attribute via a logit transform for individual i 's expected utility change from attribute level $j - 1$ to j as

$$\eta_j^i = \log\left(\frac{\alpha_j^i}{\alpha_{K+1}^i}\right) = \chi_j + \rho_{j1}z_1^i + \cdots + \rho_{jQ}z_Q^i, \quad j = 2, \dots, K,$$

where expected utility change from level K to $K + 1$ is used as the level of reference.

- Incorporating random effects

$$\eta_j^i = \chi_j + \rho_{j1}z_1^i + \cdots + \rho_{jQ}z_Q^i + \epsilon_i$$

- Computations via MCMC

ILLUSTRATION USING HUI2 BASED DATA

- Data from McCabe, Stevens and Brazier (2005).

The data is from an HUI2 based survey conducted on $N = 201$ individuals drawn from the general population of UK. The sample was stratified by mainland UK socio-economic regions based on the 1991 survey.

HUI2 is originally developed using seven attributes to describe an individual's health state. One of these original attributes, fertility, is not included in this survey.

The attributes self care, sensation and cognition has four levels while mobility, emotion and pain has five levels representing different states of these attributes.

- The levels of the attributes are structured so that preference ordering is monotonic.
- Used flat but proper priors for all coefficients in the analysis.

DESCRIPTIVE STATISTICS FROM DATA

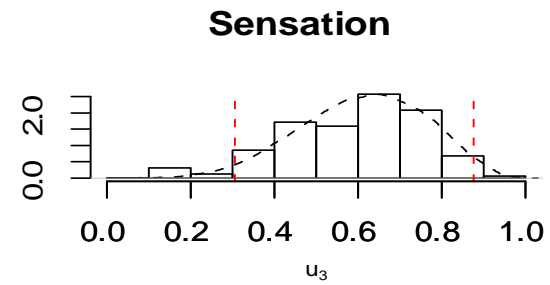
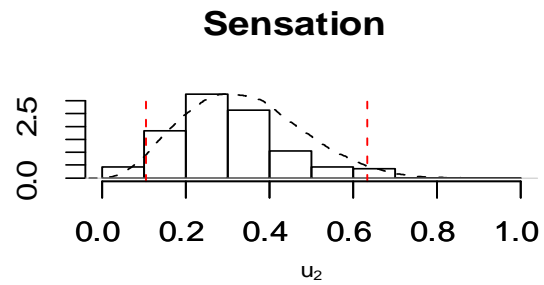
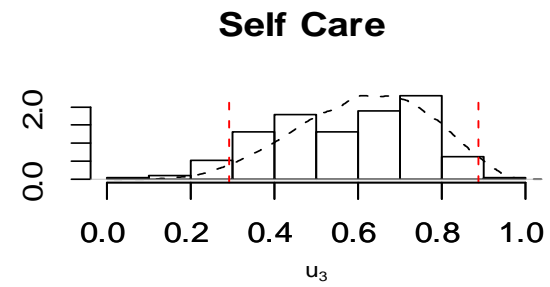
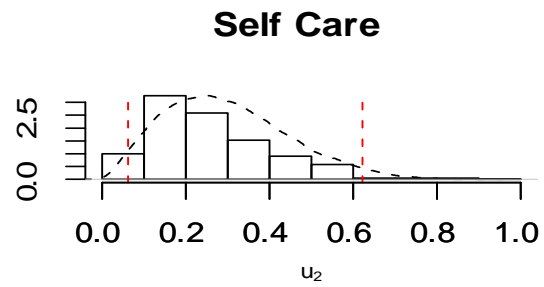
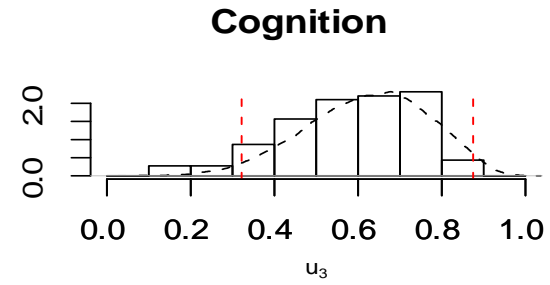
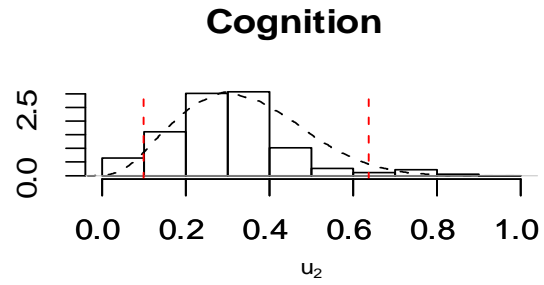
Table 1: Descriptive statistics of utilities for different attribute levels.

	<i>Sensation</i>		<i>Mobility</i>			<i>Emotion</i>			<i>Cognition</i>		<i>SelfCare</i>		<i>Pain</i>		
	u_2	u_3	u_2	u_3	u_4	u_2	u_3	u_4	u_2	u_3	u_2	u_3	u_2	u_3	u_4
<i>Mean</i>	.34	.63	.26	.48	.74	.19	.39	.64	.34	.62	.29	.61	.22	.46	.74
<i>Median</i>	.30	.70	.20	.50	.80	.20	.40	.70	.30	.60	.30	.60	.20	.45	.80
<i>Stdev</i>	.10	.15	.12	.13	.11	.04	.07	.10	.10	.10	.01	.02	.02	.10	.15

Table 2: Descriptive statistics of the scaling coefficients.

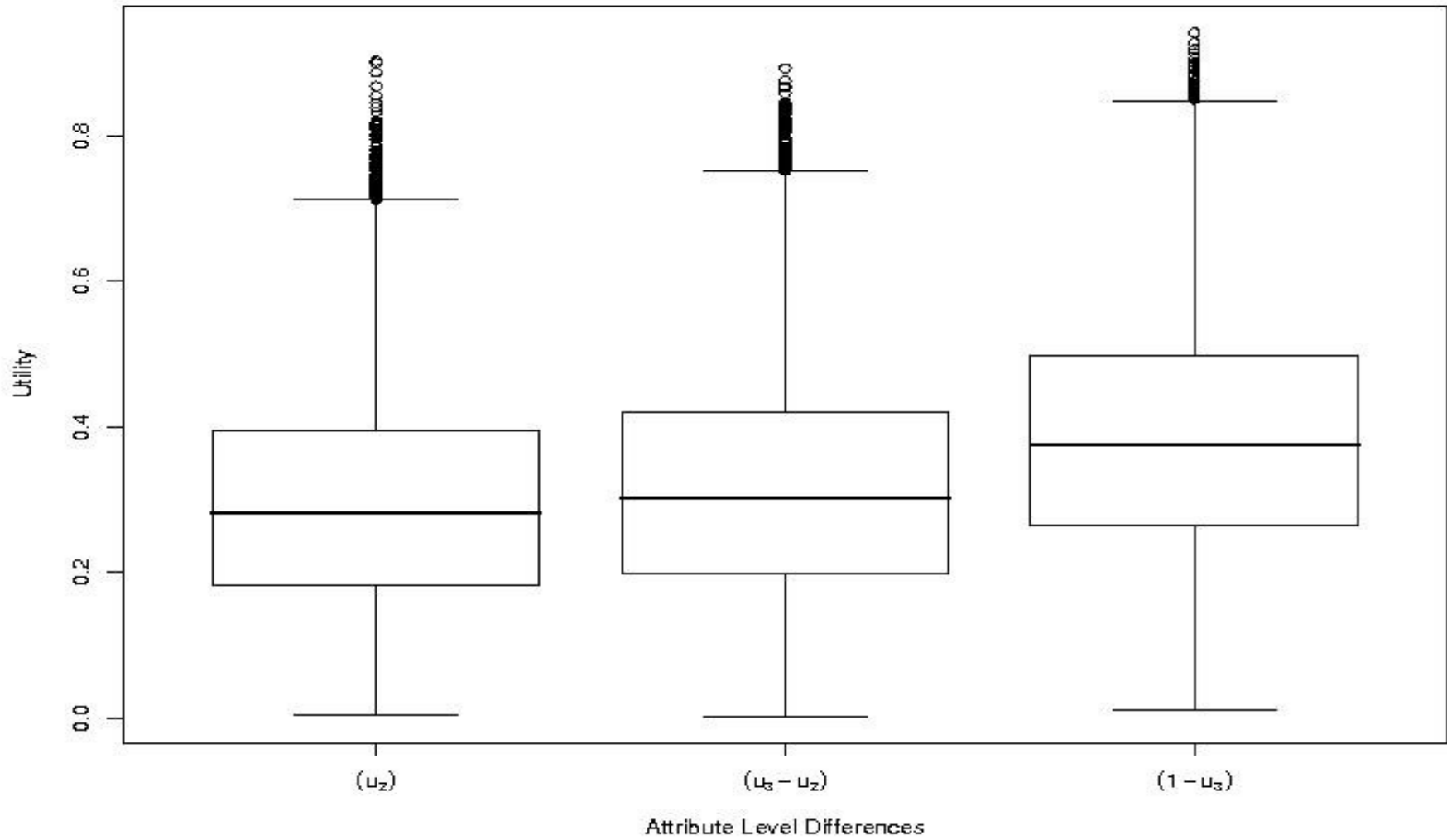
	<i>Sensation</i>	<i>Mobility</i>	<i>Emotion</i>	<i>Cognition</i>	<i>SelfCare</i>	<i>Pain</i>	k
<i>Mean</i>	.512	.449	.628	.210	.303	.407	– .927
<i>Median</i>	.500	.427	.650	.200	.271	.400	– .963
<i>Stdev</i>	.254	.213	.198	.154	.241	.225	.104

POSTERIOR PREDICTIVE FIT TO OBSERVED UTILITY



POSTERIOR PREDICTIVE UTILITY DIFFERENCE

Posterior Predictive Utility of Self Care



PROBABILITY OF RISK ATTITUDES

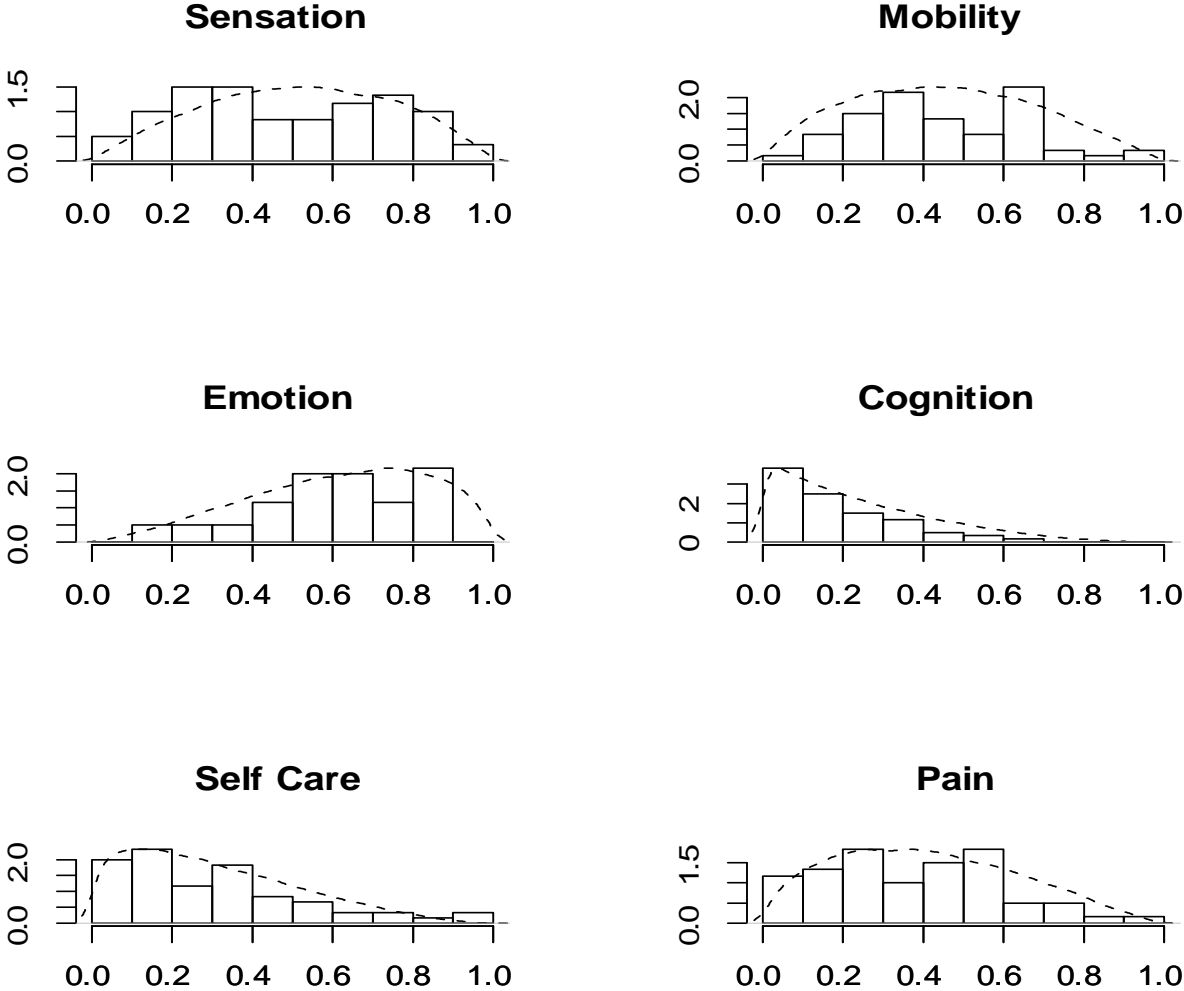
Increasing utility differences ?

Decreasing utility differences ?

Probability of utility differences in self care attribute.

	Model	Data
$P(u_3 - u_2 \geq u_2, 1 - u_3 \geq u_3 - u_2)$.237	.199
$P(u_3 - u_2 \leq u_2, 1 - u_3 \leq u_3 - u_2)$.106	.063
$P(u_3 - u_2 \leq u_2, 1 - u_3 \geq u_3 - u_2)$.365	.408
$P(u_3 - u_2 \geq u_2, 1 - u_3 \leq u_3 - u_2)$.291	.330

POSTERIOR PREDICTIVE DISTRIBUTIONS OF SCALING COEFFICIENTS



OVERALL GOODNESS OF FIT TO HEALTH STATE UTILITIES

Percentiles of the sample and predictive distributions for different health states

Health States										
	1,3,1,1,1,1		1,4,2,1,1,1		1,1,3,2,1,1		3,3,2,3,3,2		3,3,4,4,4,4	
%	Sample	Model	Sample	Model	Sample	Model	Sample	Model	Sample	Model
5%	.350	.351	.200	.253	.200	.316	.050	.061	.000	.000
25%	.600	.535	.313	.403	.308	.468	.150	.140	.050	.029
50%	.750	.676	.500	.523	.500	.582	.200	.210	.095	.057
75%	.800	.805	.600	.644	.688	.692	.315	.293	.118	.109
95%	.900	.926	.750	.789	.850	.831	.478	.429	.300	.217

POSTERIOR PROBABILITY OF HEALTH STATE PREFERENCES

Health states $\mathbf{X}_1 = (1, 1, 1, 3, 1, 2)$, $\mathbf{X}_2 = (1, 1, 3, 2, 1, 1)$, and $\mathbf{X}_3 = (1, 4, 3, 1, 1, 1)$.

$P(X_1 \succ X_2)$.658
$P(X_2 \succ X_3)$.795
$P(X_1 \succ X_3)$.794
$P(X_1 \succ X_2 \succ X_3)$.509
$P(X_2 \succ X_1 \succ X_3)$.170
$P(X_3 \succ X_1 \succ X_2)$.033
$P(X_1 \succ X_3 \succ X_2)$.116
$P(X_2 \succ X_3 \succ X_1)$.117
$P(X_3 \succ X_2 \succ X_1)$.055

OUT OF SAMPLE PREDICTIONS

- Using composite utility data (10 health states) which was not included in our analysis
- Model's estimate of mean utility of each health state versus actual mean utility \bar{U} .

Health State	Actual \bar{U}	$E(\bar{U})$	$\bar{U} - E(\bar{U})$	$\bar{U} - E(\bar{U}) / \bar{U}$
4,1,1,1,1,1	0.474	0.483	0.0095	0.02014
1,3,1,1,1,1	0.653	0.659	0.0063	0.00959
1,1,5,1,1,1	0.395	0.384	0.0105	0.02669
1,1,1,3,1,1	0.739	0.842	0.1032	0.13958
1,5,1,1,4,1	0.488	0.371	0.1166	0.23892
1,1,1,1,1,4	0.559	0.679	0.1203	0.21514
1,4,2,1,1,1	0.503	0.519	0.0165	0.03278
1,1,3,2,1,1	0.518	0.578	0.0607	0.11726
3,3,2,3,3,2	0.241	0.223	0.0180	0.07476
3,3,4,4,4,4	0.103	0.067	0.0357	0.34621

COMPARISON WITH OTHER APPROACHES

Out of sample prediction comparison of three models using the composite utility data of 10 health states:

- Our parameteric Bayes model has a MAE of 0.049 and a MAPE error of 0.122
- Classical parametric model of McCabe et al. (2005) has a MAE of 0.072 and MAPE of 0.166.
- Nonparametric Bayesian model of Kharroubi and McCabe (2008) has MAE of 0.041 and MAPE of 0.099.

SUMMARY OF FINDINGS

- The parametric Bayes model with the multiplicative utility function with ordered Dirichlet model on attribute utilities and the independent beta models on the attribute coefficients provides adequate fit to actual composite health state utility data.
- There is strong evidence in favor of a multiplicative model rather than an additive model as considered in previous parametric approaches.
- Our analysis with the covariate effects showed potential gender differences in preference of health states as well as age differences with respect to attribute weights.
- The parametric Bayesian model has better out-of-sample predictive performance than the classical parametric models and has comparable predictive performance to the nonparametric Bayesian models.