

Construction of Leading Economic Index for Recession Prediction using Vine Copulas

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Abstract

This paper constructs a composite leading index for business cycle prediction based on vine copulas that capture the complex pattern of dependence among individual predictors. This approach is optimal in the sense that the resulting index possesses the highest discriminatory power as measured by the receiver operating characteristic (ROC) curve. The model specification is semi-parametric in nature, suggesting a two-step estimation procedure, with the second-step finite dimensional parameter being estimated by QMLE given the first-step non-parametric estimate. To illustrate its usefulness, we apply this methodology to optimally aggregate the ten leading indicators selected by The Conference Board (TCB) to predict economic recessions in the United States. In terms of both the in-sample and out-of-sample performances, our method is significantly superior to the current Leading Economic Index proposed by TCB.

JEL Classifications: C14, C15, C32, C43, C51, C53, E37

Key words: Leading Economic Index, Receiver operating characteristic curve, Vine copula, Block bootstrap.

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1 Introduction

The periodic up-and-down movements in aggregate economic activities, known as business cycles, are of central importance to diverse decision-makings. The need for an accurate and timely forecast of future economic conditions arises in the current decisions of manufacturers, investors, financial intermediaries, and central banks alike. Inspired by the seminal work of Burns and Mitchell (1946), a great body of literature is devoted to prediction of the underlying evolving state of the real economy. These can be essentially classified into two strands. The first approach, with the simple autoregressive model as a typical example, is to employ the historical information of the past cycles to predict future based on the persistence exhibited by most economic series. The second one focuses on the contemporaneous information embodied in some appropriately selected indicators. As characterized by Burns and Mitchell (1946), “business cycles ... [which] consist of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions and revivals which merge into the expansion phase of the next cycle”. These economic activities, as measured by various macroeconomic indicators, would evolve over time along with the unobserved cycles. The fact that the statistical properties of these indicators are different when the regime of the economy switches provides grounds for the second approach. Unlike the first one, this approach does not assume the continuity of the underlying economic series, which renders it more attractive for turning point prediction. In this paper, we will follow the second approach by constructing a composite index by extracting the predictive information contained in a few representative leading indicators.

One of the most prominent indices in the United States is the Leading Economic Index (LEI) developed by The Conference Board (TCB). Rather than being model-based, it simply summarizes 10 leading indicators in a straightforward fashion. Since it is easy to calculate and understand, the LEI enjoys great popularity in business cycle forecasting. However, due to its lack of a statistically rigorous foundation, we are not aware if it is optimal in some sense. To overcome this drawback, we create a new index that combines the very 10 leading indicators within a probabilistic framework. Unlike the current LEI, our index has

an interpretation of recession probability. It can be proved to be optimal to maximize the predictive accuracy as measured by the receiver operating characteristic (ROC) curve. The ROC curve is designed to evaluate the discriminatory capacity of a forecasting system to predict a binary event. Compared with the commonly used mean squared error, the ROC curve is not influenced by the marginal distribution of the target variable, making it more adequate to use in forecasting a relatively uncommon event, like economic recession. This methodology was initially developed in the field of signal detection theory, where it was used to evaluate the discriminatory ability for a binary detection system to distinguish between two clearly-defined possibilities: signal plus noise and noise only. Thereafter, it has gained increasing popularity in many other related fields. A general introduction to ROC analysis can be found in Fawcett (2006), Pepe (2000), Swets et al. (2000), and Zhou et al. (2002). Until recently, ROC has not gained much attention from econometricians. Exceptions include Berge and Jordà (2011), Drehmann and Juselius (2014), Jordà and Taylor (2012), Lahiri and Wang (2013), Liu and Moench (2014), just to name a few.

Our leading index depends on copula functions, a method dating back to 1959 paper by Sklar. The optimal way to aggregate the 10 indicators can be shown to be a non-linear functional of the conditional distribution of these predictors given the actual. One might use a fully specified parametric model, such as the multivariate normal distribution. However, it is subject to misspecification bias. Alternatively, we might not impose any constraint and use a non-parametric approach to estimate the conditional distribution. However, it suffers from the “curse of dimensionality” because the sample size is not large enough when compared with the number of predictors. The approach we take is semi-parametric in nature. The marginal distribution of each predictor is estimated non-parametrically, while the dependence parameters amongst predictors that are represented by copulas are estimated by quasi-maximum likelihood. On the one hand, this approach allows for a larger degree of flexibility in specifying the marginal distributions than the full parametric approach. On the other hand, the dimension of non-parametric estimation is always one so that the “curse of dimensionality” is resolved. The only difficulty comes from the specification of high-dimensional copulas. To capture the realistic yet complex dependence structure in a high-dimensional setting, we cannot use the direct extension of regular bivariate copulas since they require homogene-

ity in dependence. As an alternative, this paper develops multivariate distributions of the 10 predictors based on vine copulas, which rely on bivariate copulas as building blocks. It is recognized that knowing the high-dimensional dependence is harder than knowing the bivariate dependence. Thus, our method is easy-to-use as long as a set of bivariate copulas are chosen appropriately. A two-step procedure is advocated to estimate the unknown quantities in our model, with each step being computationally affordable in a standard package. In the empirical application, our composite index can deliver a better classification of all time periods into the two regimes than the current LEI both in- and out-of-sample.

The paper is organized as follows. In Section 2, we construct the composite index by combining the multiple predictors based on the vine copulas and we show this combination rule is optimal to maximize the ROC curve. A two-step estimation procedure is also described. Section 3 illustrates the usefulness of our method in predicting economic recessions in the United States using series of the 10 leading indicators of TCB. Finally, in Section 4, we offer concluding remarks and suggestions of further extensions.

2 A copula-based combination scheme

Throughout this section, we denote the individual predictor by X_i for $i = 1, 2, \dots, I$, and the binary variable to be predicted is Z , which is one when the event of interest occurs and zero otherwise. We use upper case letters to denote cumulative distribution functions and corresponding lower case letters to denote the density functions. Our objective is to use the information content in each X_i to predict the occurrence of the binary event. For this purpose, forecast combination is a natural way to pursue. Let X be the vector of $(X_1, X_2, \dots, X_I)'$. Given Z , we have two conditional distributions of X , namely, $F(\cdot|Z = 1)$ and $F(\cdot|Z = 0)$, both of which are I -dimensional cumulative distribution functions. Before outlining our combination scheme, it is necessary to discuss the modeling of $F(\cdot|Z = 1)$ and $F(\cdot|Z = 0)$, which are two building blocks of the scheme.

The simplest case to deal with is that X follows the multivariate normal distribution given Z . In this scenario, the optimal combined forecast takes an interesting form, as shown later.

Despite its parsimony and familiarity, the normality assumption has several key shortcomings. First, it requires that each X_i be normally distributed. Thus, it rules out those predictors, whose distributions are skewed, fat-tailed, or multimodal. Furthermore, the discrete predictor is not allowed. Second, the dependence structure among predictors is also restricted. In the literature of multivariate statistics, more than one dependence measures exist. The most commonly used is the Pearson correlation coefficient. The multivariate normal distribution is flexible in terms of this particular measure since it permits any type of correlation matrix as long as it is positive definite. However, the Pearson correlation is affected by the marginal distributions of the predictors, and thereby is not regarded as the true dependence measure (Joe (1997)). Moreover, it merely measures the dependence in the center of the distributions. In practice, interest may center on the dependence behavior in the left and right tails of the distributions. As an illustration, we consider the first two predictors, i.e. X_1 and X_2 . The upper tail dependence coefficient is defined as $\lim_{q \rightarrow 1^-} P(X_2 > F_2^{-1}(q) | X_1 > F_1^{-1}(q))$, and the lower tail dependence coefficient is $\lim_{q \rightarrow 0^+} P(X_2 \leq F_2^{-1}(q) | X_1 \leq F_1^{-1}(q))$, where $F_j^{-1}(\cdot)$ is the quantile function of X_j for $j = 1, 2$. Roughly speaking, both coefficients are the probability that one predictor is very large (small) given that the other is also very large (small). As a result, they measure the dependence between X_1 and X_2 when both are extremely large (small). Many predictors in X may exhibit strong tail dependence in practice. Unfortunately, the normality assumption does not entail positive dependence in both tails so that extreme events appear to be uncorrelated, see Demarta and McNeil (2005) for more details. For this reason, we will not impose multivariate normality. The approach we are going to take is much more robust and it remedies the two aforementioned pitfalls.

The first pitfall is concerned with the non-normality of the marginal distributions. In our empirical application, each predictor in X reflects one aspect of the whole economy, which is related to future economic activities. The Conference Board uses different transformations to construct these predictors. For example, TCB computes the symmetric percentage change of the average weekly hours, but it sticks with the level form of the interest rate spread. These transformations are likely to change the marginal distributions of some predictors, even though the original series can be represented by a simple data generating process (DGP). It is hard, if not impossible, to figure out the implied distribution of the transformed predictor

when the DGP of the original series is known. Following the same line of Harding and Pagan (2011), we will not impose any additional distributional restriction on each X_i beyond some smoothness conditions. Instead, the marginal distributions are estimated by non-parametric method. Specifically, let $k(\cdot)$ be a second-order symmetric kernel function. The marginal densities of X_i given $Z = 1$ and $Z = 0$ are estimated by

$$\hat{f}_i^1(x_i) \equiv \frac{1}{h_i^1 \sum_{t=1}^T I(Z_t = 1)} \sum_{t=1}^T I(Z_t = 1) k\left(\frac{x_i - X_{it}}{h_i^1}\right),$$

and

$$\hat{f}_i^0(x_i) \equiv \frac{1}{h_i^0 \sum_{t=1}^T I(Z_t = 0)} \sum_{t=1}^T I(Z_t = 0) k\left(\frac{x_i - X_{it}}{h_i^0}\right), \quad (1)$$

respectively, where $I(\cdot)$ is the indicator function which equals one when the condition in the parenthesis is true and zero otherwise. h_i^1 (when $Z = 1$) and h_i^0 (when $Z = 0$) are the bandwidths for X_i chosen by the researcher. The choice of h_i depends upon the tradeoff between the bias and the variance of the resulting estimator and the selected bandwidth may vary across i . A large body of literature discusses the optimal choice of h_i , see Li and Racine (2008) for a comprehensive review.

To address the second pitfall, we adopt copula functions. Since the seminal work of Sklar (1973), the multivariate modeling based on copulas has received growing attention in statistics. A copula is a multivariate distribution function whose univariate marginals are uniforms between zero and one. For any I -dimensional distribution function $F(x_1, x_2, \dots, x_I)$ with $F_i(\cdot)$ as its marginal distribution, the usefulness of copula roots in the following decomposition theorem, i.e.,

$$F(x_1, x_2, \dots, x_I) = C(F_1(x_1), F_2(x_2), \dots, F_I(x_I)), \quad (2)$$

for all $(x_1, x_2, \dots, x_I) \in R^I$, where C is the copula associated with $F(x_1, x_2, \dots, x_I)$. If X is continuously distributed, C is uniquely determined. To uncover the corresponding copula,

we can use the inverse of (2), i.e.,

$$C(v_1, v_2, \dots, v_I) = F(F_1^{-1}(v_1), F_2^{-1}(v_2), \dots, F_I^{-1}(v_I)), \quad (3)$$

where $F_i^{-1}(\cdot)$ is the inverse distribution function of X_i . Both (2) and (3) provide a wise way of isolating marginal distributions with their dependency structure. One can model $F_i(\cdot)$ individually, and then choose a reasonable copula C to form a joint distribution. This theorem assures that $F(x_1, x_2, \dots, x_I)$ resulting from (2) is a valid multivariate distribution function. A general introduction to the modeling strategies based on copulas was given by Joe (1997), Nelsen (2006), Patton (2012), and Trivedi and Zimmer (2005). Anatolyev (2009), Patton (2006), Patton (2013) and Scotti (2011) applied this methodology to predict multiple economic events. Patton and Fan (2014) provided a recent survey on copula methods from the perspective of econometrics.

Like marginal distributions, the copula can be estimated non-parametrically. However, the “curse of dimensionality” argument implies that the estimate is not reliable if the number of predictors, I , exceeds two and the sample size T is not large enough. Unfortunately, this is the case in our empirical application, where we have a sample of moderate size with 10 predictors in X . Chen and Fan (2006) considered a semi-parametric copula-based model and developed a two-step estimation procedure. In the first step, the marginal distributions of standardized innovations are estimated non-parametrically. In the second step, the parameters in the copula are estimated by quasi-maximum likelihood when each marginal in the likelihood function is replaced by the first-step estimate. Like other semi-parametric models, Chen and Fan’s approach avoids the “curse of dimensionality” in that the multivariate component, i.e. the copula, is parameterized. In order to apply this methodology in our context, a parametric family of copula must be specified. The analysts often know little about the dependence structure though they might be pretty sure of the marginal distributions. Therefore, selecting an appropriate copula seems to be quite challenging especially in a high-dimensional setting. When $I = 2$, there are a large number of bivariate candidates. These include, but are not limited to, elliptical class (Gaussian copula and t copula), Archimedean families (Clayton copula, Gumbel copula, Frank copula, etc.), and the finite mixture of bivariate copulas. Many

graphic and analytical goodness-of-fit tests are proposed to provide guidelines in specifying an adequate bivariate copula, see Chen and Fan (2006,2007), Fermanian (2005), Genest *et al.* (2009), Klugman and Parsa (1999), just to name a few.

When $I > 2$, the parametric families of copulas are very scarce. The most obvious choice is multivariate elliptical class. The I -dimensional Gaussian copula can be generated by (3) if F is the I -dimensional normal distribution function. Alternatively, we can replace F by the I -dimensional t distribution to obtain the multivariate t copula. However, these higher dimensional extensions cannot accommodate the complicated tail dependence structures among different pairs of predictors. For example, the multivariate t copula has only one degree of freedom, meaning that all pairs of predictors share the same tail dependence pattern, which is not realistic. In this paper, we exploit the recent advances on vine copulas (Aas *et al.* (2009), Berg and Aas (2009), Czado (2010), Kurowicka and Joe (2011)) since they are constructed from the bivariate copulas, which are much easier to specify and estimate.

To fix the idea, we consider a trivariate density, namely, $f_{123}(x_1, x_2, x_3)$. First, $f_{123}(x_1, x_2, x_3)$ can be decomposed into the product of one marginal density and two conditional densities,

$$f_{123}(x_1, x_2, x_3) = f_1(x_1)f_{2|1}(x_2|x_1)f_{3|12}(x_3|x_1, x_2). \quad (4)$$

It follows by twice differentiating (2) for $I = 2$ that

$$f_{12}(x_1, x_2) = f_1(x_1)f_2(x_2)c_{12}(F_1(x_1), F_2(x_2)), \quad (5)$$

where $c_{12}(v_1, v_2) \equiv \partial^2/\partial v_1\partial v_2 C_{12}(v_1, v_2)$ is the copula density between X_1 and X_2 . This implies that

$$f_{2|1}(x_2|x_1) = \frac{f_{12}(x_1, x_2)}{f_1(x_1)} = f_2(x_2)c_{12}(F_1(x_1), F_2(x_2)). \quad (6)$$

By the same reasoning, we have

$$f_{3|12}(x_3|x_1, x_2) = \frac{f_{3|2}(x_3|x_2)f_{1|23}(x_1|x_2, x_3)}{f_{1|2}(x_1|x_2)} = f_{3|2}(x_3|x_2)c_{13|2}(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)), \quad (7)$$

where $c_{13|2}$ is the copula between X_1 and X_3 given X_2 . Substituting (6) and (7) into (4), $f_{123}(x_1, x_2, x_3)$ can be expressed as

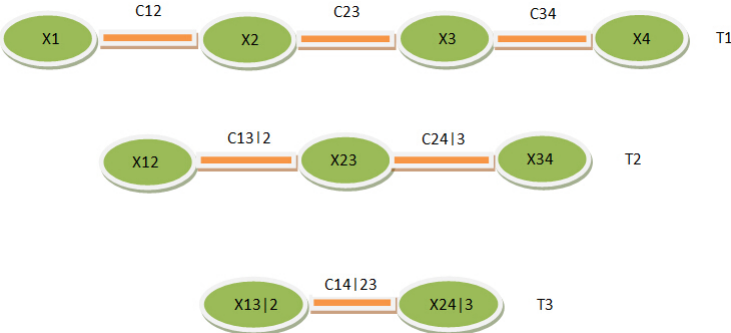
$$f_1(x_1)f_2(x_2)f_3(x_3)c_{12}(F_1(x_1), F_2(x_2))c_{23}(F_2(x_2), F_3(x_3))c_{13|2}(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)), \quad (8)$$

where we make use of the fact that $f_{3|2}(x_3|x_2) = f_3(x_3)c_{23}(F_2(x_2), F_3(x_3))$. Consequently, the trivariate density can be reformulated as the product of three marginal densities $(f_1(x_1), f_2(x_2), f_3(x_3))$ and three bivariate copula densities $(c_{12}, c_{23}, c_{13|2})$. Two copulas are unconditional (c_{12}, c_{23}) and one is conditional $(c_{13|2})$. In general, any I -dimensional density can be decomposed in this way. There are I marginal densities, $I - 1$ unconditional copulas and $(I - 2)(I - 1)/2$ conditional copulas in the decomposition. The readers are referred to Aas *et al.* (2009) for the general formula when $I > 3$.

(8) is called pair-copula construction (PCC) of multivariate distribution. For high-dimensional densities, we can generate different decompositions by changing the ordering of the predictors. For example, there are totally 240 different constructions of a 5-dimensional density. Bedford and Cooke (2001) suggested the use of a nested tree structure to visualize a specific pair-copula construction and called it *the regular vine*. Among the class of the regular vine, we only focus on the so called *D-vine* (Kurowicka and Cooke (2004)), where all predictors are treated equally and none of them plays a central role. A specific example of the D-vine with four predictors is illustrated in Figures 1. The first tree T_1 has four nodes, each corresponding to an individual predictor. The edge connecting the adjacent nodes represents the bivariate unconditional copula describing the dependence between these two nodes. In the second tree T_2 , there are only three nodes, each of which corresponds to the edge in the previous tree T_1 . The edges in T_2 represent the bivariate conditional copulas with the common predictor between the adjacent nodes being the conditioning variable. The last tree T_3 has two nodes, which are linked by one edge. The conditioning variables in the copula are given by the union of all conditioning variables in the two nodes. The whole vine is characterized by three trees, nine nodes and six edges. Note that the D-vine structure in Figures 1 is uniquely determined once the ordering of the predictors in T_1 is fixed. The question thus boils down to how the ordering in T_1 is determined. To the best of our knowledge, there is no formal

strategy in the literature to specify the ordering given a dataset. In practice, this may depend on which bivariate relationships among predictors are the most important to model. Suppose X_1 is more correlated with X_3 than it is with X_2 in the sample. It is better to put X_1 closer to X_3 than to X_2 . Aas *et al.* (2009) provided an empirical example in finance to implement this strategy in a four-dimensional case. For a high-dimensional illustration, the readers are referred to Dissmann *et al.* (2013).

Figure 1: A D-vine with 4 variables, 3 trees and 6 edges



The idea of PCC was first explored by Joe (1996) and Bedford and Cooke (2001, 2002). Aas *et al.* (2009) established the main framework and proposed a simulation algorithm, as well as a sequential estimation method of PCC model. However, this methodology has not received much attention by econometricians to date. Both the theoretical and the empirical papers on PCC are scarce. Exceptions may include Brechmann and Czado (2013), Minand Czado (2010), and Zimmer (2014). Compared with the multivariate elliptical copulas, PCC model provides more flexibility because it enables us to specify a wide range of parametric families of copulas to capture the different (conditional or unconditional) dependence structures in Figures 1. These pair-copulas do not necessarily belong to the same parametric family. For example, the empirical evidences show that X_1 and X_2 are independent of each other in both tails, while a strong upper tail dependence exists between X_2 and X_3 (but weak lower tail dependence). The Gaussian copula may be suitable for c_{12} , and the Gumbel copula is a good choice for c_{23} . Nevertheless, bivariate copula specification is essential to make PCC model more useful. Many goodness-of-fit tests would help in choosing a series of copulas that best fit the data. As a cost, the PCC model often includes many parameters to estimate, especially when I is large. Fortunately, we do not need to estimate all parameters in one step.

Instead, we can estimate the marginal distributions by (1), and then estimate the parameters in each bivariate copula using the algorithm given by Aas *et al.* (2009). Though inefficient by themselves, these sequential estimators are consistent and asymptotically normally distributed, as shown by Chen and Fan (2006). In order to achieve asymptotic efficiency, we can take these sequential estimates as the starting values, and maximize the likelihood function by a recursive algorithm. The joint maximum likelihood estimator is expected to be efficient.

The main goal of this paper is to construct a new composite index summarizing the idiosyncratic information contained in each indicator in an optimal way. A range of forecast skill measures have been proposed in the literature to quantify the performance of competing forecasts. Here, we use the receiver operating characteristic (ROC) curve and the area under the curve (AUC). ROC visualizes the discriminatory power of a forecasting system in distinguishing between $Z = 1$ and $Z = 0$. If the forecasts are completely insensitive to the value Z would take, they have zero discriminatory power. On the other hand, forecasts which take one value when $Z = 1$ and take another when $Z = 0$ would obviously possess the highest discriminatory power. Most real-life forecasts lie between these two extremes. Without loss of generality, a cutoff w is adopted such that an observation with $X_1 > w$ is predicted as $Z = 1$. We can define two conditional probabilities resulting from this decision rule, namely,

$$\begin{aligned} H(w) &\equiv P(X_1 > w | Z = 1), \\ F(w) &\equiv P(X_1 > w | Z = 0). \end{aligned} \tag{9}$$

$H(w)$ is referred to as the hit rate and it is the probability of correct forecast when $Z = 1$. $F(w)$ is called the false alarm rate or the probability of false forecast when $Z = 0$. Ideally, we hope $H(w)$ could be as large as possible and $F(w)$ should be as small as possible. Both of them are functions of w . In general, given the forecasting system, it is hard to achieve a high value of $H(w)$ without changing $F(w)$. The tradeoff between them is depicted by plotting the pair $(F(w), H(w))$ in a unit square for a fine grid of w . The resulting ROC curve is an increasing function from $(0,0)$ to $(1,1)$. The ROC curve for forecasts with zero discriminatory power is represented by the diagonal line in the unit square with its AUC 0.5. Conversely, the ROC curve described by the left and upper boundaries of the square has the

highest discriminatory power with its AUC 1. Most real-life forecasts yield an ROC curve lying in the upper triangular area whose AUC is strictly between 0.5 and 1.

When more than one predictors are available, none of them could be maximally utilized unless the information contained in them is processed in an efficient manner so that the hit rate is maximized for any given false alarm rate. The region C_α defined as

$$\{X : \frac{f(X|H_1)}{f(X|H_0)} > w\}$$

plays a critical role in testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$. Here, $X = (X_1, X_2, \dots, X_I)'$ is the vector of all predictors, $f(X|H_j)$ is the likelihood function under H_j for $j = 0, 1$, and w is a constant such that $P(f(X|H_1)/f(X|H_0) > w|H_0) = \alpha$. Among all tests of H_0 against H_1 with the same size α , Neyman-Pearson lemma states that the power, defined as $P(f(X|H_1)/f(X|H_0) > w|H_1)$, achieves its maximum if we reject H_0 when $f(X|H_1)/f(X|H_0) > w$. Therefore, the likelihood ratio test for simple hypothesis is the most powerful. The implication of this lemma is that the rule constructed from the likelihood ratio of multiple predictors maximizes the hit rate for any given false alarm rate. This rule can be used to justify the test $H_0 : Z = 0$ against $H_1 : Z = 1$, where both the null and the alternative are simple. When $I = 3$, the combined forecast takes the following form,

$$\frac{\hat{f}_1^1(x_1)\hat{f}_2^1(x_2)\hat{f}_3^1(x_3)\hat{c}_{12}^1(\hat{F}_1^1(x_1), \hat{F}_2^1(x_2))\hat{c}_{23}^1(\hat{F}_2^1(x_2), \hat{F}_3^1(x_3))\hat{c}_{13|2}^1(\hat{F}_{1|2}^1(x_1|x_2), \hat{F}_{3|2}^1(x_3|x_2))}{\hat{f}_1^0(x_1)\hat{f}_2^0(x_2)\hat{f}_3^0(x_3)\hat{c}_{12}^0(\hat{F}_1^0(x_1), \hat{F}_2^0(x_2))\hat{c}_{23}^0(\hat{F}_2^0(x_2), \hat{F}_3^0(x_3))\hat{c}_{13|2}^0(\hat{F}_{1|2}^0(x_1|x_2), \hat{F}_{3|2}^0(x_3|x_2))}. \quad (10)$$

In (10), $\hat{f}_i^j(x_i)$ is given by (1), and $\hat{F}_i^j(x_i) = \int_{-\infty}^{x_i} \hat{f}_i^j(u)du$ for $i = 1, 2, 3$ and $j = 0, 1$. All \hat{c} 's denote the copulas with the parameters being replaced by their joint maximum likelihood estimates, as mentioned before.¹ According to Joe (1996), all conditional distribution functions can be derived by the partial derivative of the selected copulas. For example,

$$F_{1|2}^j(x_1|x_2) = \frac{\partial C_{12}^j(F_1^j(x_1), F_2^j(x_2))}{\partial F_2^j(x_2)}, \quad (11)$$

¹In general, $c_{13|2}$ is a conditional copula whose parameters depend on x_2 in an unknown way. To simplify the analysis, we will avoid this complexity and assume all parameters in $c_{13|2}$ are constants. See Acar *et al.* (2011, 2012) for details on a general PCC model.

and $\hat{F}_{1|2}^j(x_1|x_2)$ denotes the estimate of (11) when all unknown objects in (11) are substituted by their estimates. We predict $Z = 1$ if and only if (10) exceeds w . In this case, the size is the false alarm rate $F(w)$, while the power is the hit rate $H(w)$. Given $F(w)$, $H(w)$ is maximized among all possible combination rules based on (X_1, X_2, \dots, X_I) . The optimality of (10) was also established in McIntosh and Pepe (2002).

The likelihood ratio in (10) is of some interest in itself. However, a more interpretable statistic is the predictive probability of $Z = 1$ given all predictors, namely, $P(Z = 1|X)$. We predict $Z = 1$ if and only if $P(Z = 1|X)$ exceeds a threshold value. An ROC curve can be generated by using $P(Z = 1|X)$ instead of (10). Lahiri and Yang (2015) showed that both ROC curves are identical since $P(Z = 1|X)$ is a strictly increasing transformation of (10) due to Bayes' theorem. Note that

$$\begin{aligned} P(Z = 1|X) &= \frac{f(X|Z = 1)P(Z = 1)}{f(X|Z = 0)P(Z = 0) + f(X|Z = 1)P(Z = 1)} \\ &= \frac{\frac{f(X|Z=1)}{f(X|Z=0)}P(Z = 1)}{P(Z = 0) + \frac{f(X|Z=1)}{f(X|Z=0)}P(Z = 1)}, \end{aligned} \quad (12)$$

which is obviously monotonic with the likelihood ratio $f(X|Z = 1)/f(X|Z = 0)$. Hastie et al. (2001) proved that if the two conditional distributions, i.e. $f(X|Z = 1)$ and $f(X|Z = 0)$, are I -dimensional normal², the corresponding $P(Z = 1|X)$ can be described by a logit regression model with a quadratic index. In addition, if these two normal distributions share the same covariance matrices, $P(Z = 1|X)$ reduces to a logit regression with a linear index. In other words, the restrictions imposed on (10) can be translated into those imposed on $P(Z = 1|X)$. The implied $P(Z = 1|X)$ from (10) might not take a well-known form if all marginal distributions are non-parametrically estimated and some pairwise copulas are not Gaussian.

Given a time series data $\{(Z_t, X_t) : t = 1, \dots, T\}$, we first estimate all unknown quantities in (10) using the previous procedure, and evaluate the estimated (10) at each observation. Given a threshold w , we can calculate the proportion of cases when $Z_t = 1$ is correctly predicted, which is the empirical hit rate $\hat{H}(w)$. Analogously, the empirical false alarm rate $\hat{F}(w)$ can

²This holds if the marginal distribution of each predictor given Z is normal and all pairwise copulas in (10) are Gaussian.

be calculated as well. By plotting the pair $(\hat{F}(w), \hat{H}(w))$ for a range of w , the empirical ROC curve is obtained. The empirical AUC can be derived numerically as in Fawcett (2006). All of these empirical quantities are subject to sampling variability, which should be accounted for properly in order to draw a statistically meaningful conclusion. Lahiri and Yang (2015) derived the asymptotic confidence bands for ROC curves using time series data. It is cumbersome, if not impossible, to get the asymptotic distribution of the empirical ROC curve given the above two-step estimation procedure. We favor the bootstrap due to its convenience to conduct. To mimic the serial dependence in the original sample, a circular block bootstrap is used. The block length is either fixed or can be chosen randomly from a geometric distribution. We implement the whole estimation process on each bootstrap replicate of the original sample. The confidence intervals are constructed using these bootstrap replicates of ROC curve. The details can be also found in Lahiri (2003). This method is expected to work well when T is large enough. However, without a rigorous comparison, it is unclear whether a refinement over the asymptotic distribution can be achieved by bootstrap.

3 Empirical illustration

In this section, we will demonstrate the usefulness of the methodology developed in Section 2 by constructing a new Leading Economic Index for the sake of predicting business cycles in the United States. The business cycle is not about any single variable, like GDP, or anything else. Rather, the business cycle is about the dynamics and interactions of a set of relevant variables. The comovement of many individual economic series over the cycles leads naturally to the creation of composite index, which summarizes the idiosyncratic information encompassed in each individual indicator in an easy digestible way. It agrees with Burns and Mitchell's view and is thought of as a useful tool for monitoring and forecasting the unobserved business cycles.

Currently, the widely used Leading Economic Index (LEI) proposed by The Conference Board (TCB) is based on the following 10 predictors: ISM Index of New Orders (X_1), interest rate spread (10-year Treasury bonds less federal funds) (X_2), average weekly hours (manufac-

turing) (X_3), average weekly initial claims for unemployment insurance (X_4), manufacturers' new orders (X_5), consumer goods and materials, Manufacturers' new orders (nondefense capital goods excluding aircraft orders) (X_6), building permits, new private housing units (X_7), S&P500 (X_8), Leading Credit Index (X_9), and average consumer expectations for business conditions (X_{10}). The monthly series of these predictors run from 1959:08 to 2011:12. TCB aggregates these 10 predictors by following four steps: first, compute the month-to-month symmetric percent change for each predictor; second, adjust these changes to equalize the volatility of each predictor; third, add the adjusted changes to obtain the growth rate of the composite index; finally, compute the level of the index for the current month by using the growth rate and the last month index.³ The reference variable Z is the recession indicator defined by the National Bureau of Economic Research (NBER) business cycle dating committee. It is one if the recession occurred, and zero otherwise. The sample proportion of months that were in recession is about 14.8%, indicating that it is a relatively uncommon event. Lahiri and Yang (2015) and Levanon *et al.* (2014) assessed the performance of the LEI in terms of predicting Z using a wide variety of evaluation methodologies. They found the LEI, by utilizing diverse information implicit in the 10 predictors, performed remarkably well in signalling the incoming recessions both in- and out-of-sample. Though popular by itself, this linear combination scheme is far from optimal. The primary reason is that the four-step aggregation procedure is completely isolated from the target variable. Consequently, the resulting LEI is unlikely to provide an efficient summary. Indeed, one of our aims is to show how large the improvement over the current LEI can be gained through our non-linear combination scheme. For the sake of brevity, we merely consider six month ahead forecast, that is, X_t is used to predict Z_{t+6} .

In order to implement our method, all individual series must be stationary. Some predictors can be regarded as stationary in their level form, such as the interest spread. For others, we convert them into the month-to-month symmetric percent change to achieve stationarity. Figure 2 presents the two scatter plot matrices of the 10 predictors (after stationarity transformation) during recessions and expansions. We observe the following: (i) some of the leading indicators do not seem to be normally distributed by looking at the histograms along the diag-

³In the base year 2004, the average value of LEI is fixed at 100.

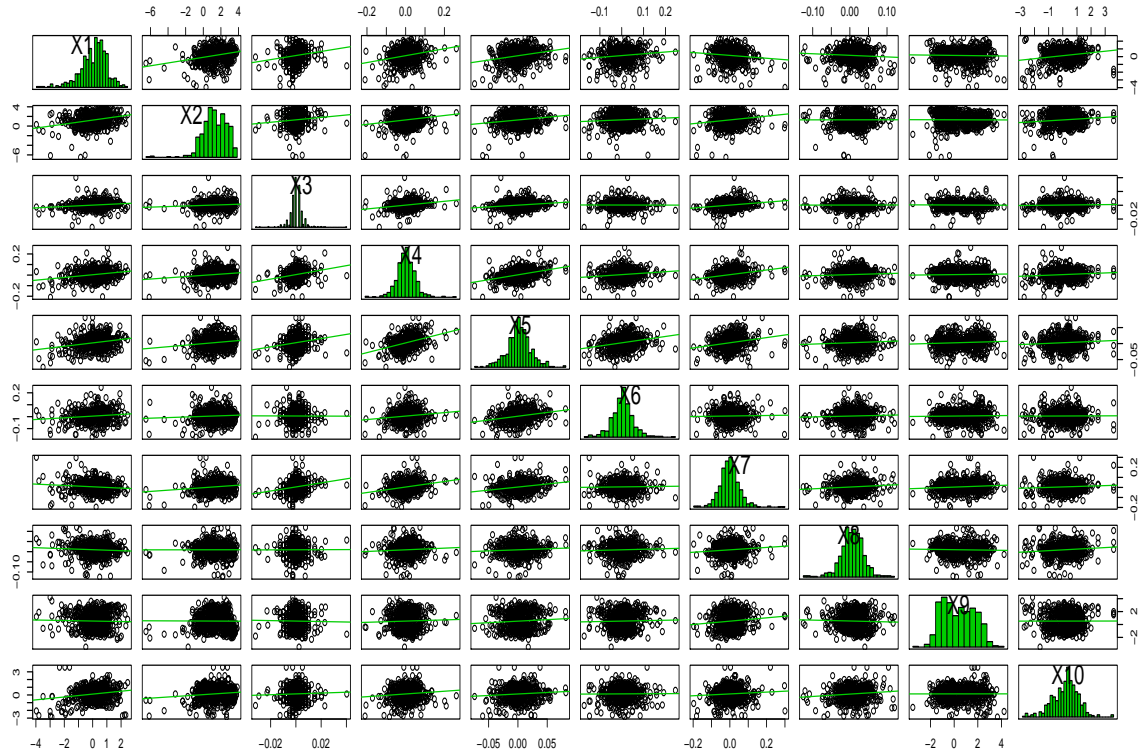
onal, especially during recessions, (ii) the dependence pattern between pairs of indicators for each regime seems to be quite different, and (iii) the correlation between a given pair seems to be different across regimes. Fortunately, our combination scheme allows for all of these features observed in the data. For each marginal distribution, we do not impose normality assumption. Instead, it is estimated by non-parametric method in (1). In a given regime, the PCC construction facilitates modeling diverse dependence structures across pairs, as argued in Section 2. Moreover, we estimate all unknown quantities for recession and expansion separately to accommodate (iii).

As for the ordering of predictors in T_1 of the D-Vine tree, we take the strategy of Aas *et al.* (2009). The principle is that the more dependent the two predictors are, the closer they should be placed in T_1 . That is, more emphasis is put on modeling stronger pair dependence, and all of other dependence structures are implied from the selected PCC. We did try other possibilities, and the results are quite robust. Having determined the ordering of predictors in T_1 , the entire tree is constructed. The next step is to choose an appropriate bivariate copula family associated with each edge. This can be handled in the R system with the aid of **CDVine** package. Brechmann and Schepsmeier (2013) provided a comprehensive introduction to various computational facilities in this package. There are 41 built-in bivariate copula families available to use, encompassing virtually all of the usual copulas that capture a wide variety of dependence patterns. Among them, the one that fits data best in terms of BIC is chosen for each pair. The complete D-Vine trees when $Z = 0$ (expansion) and when $Z = 1$ (recession) are listed in Table 1. Evidently, most of pairs are independent, and positive dependence dominates negative dependence, which is in accordance with the scatter plots in Figure 2. Note that there are 90 pair-copulas to be specified, but only 4 of them are Gaussian. In particular, the Gaussian copula is never chosen during recession. During expansion, the only two pairs whose dependence can be characterized by Gaussian copula are (X_1, X_2) and (X_4, X_5) , indicating the prevalence of asymmetric tail dependence in the data.

Figure 3 depicts the empirical ROC curve produced by our optimal procedure based on the D-Vine trees in Table 1. As comparison, we also plot the empirical ROC curve of the current LEI, as well as those generated by individual predictors. As expected, the linear composite index LEI, as a combination scheme, outperforms the best single predictor (the

Figure 2: Scatter plot matrices of the 10 predictors in two regimes

(a) Expansion



(b) Recession

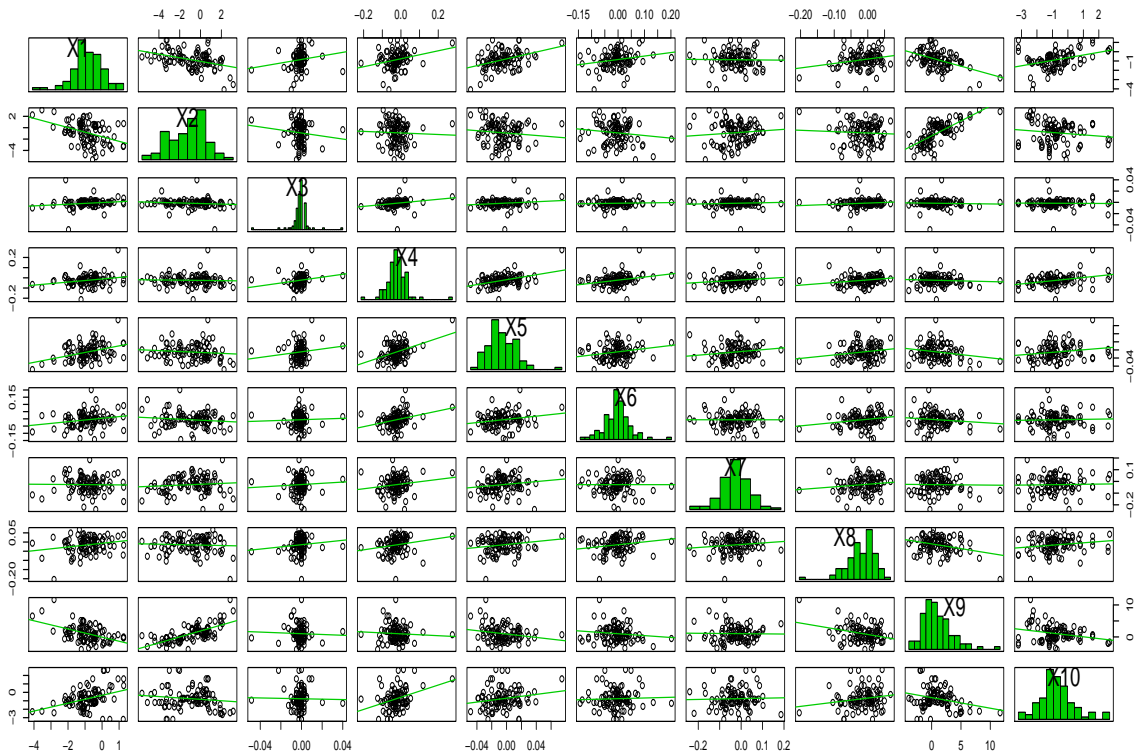


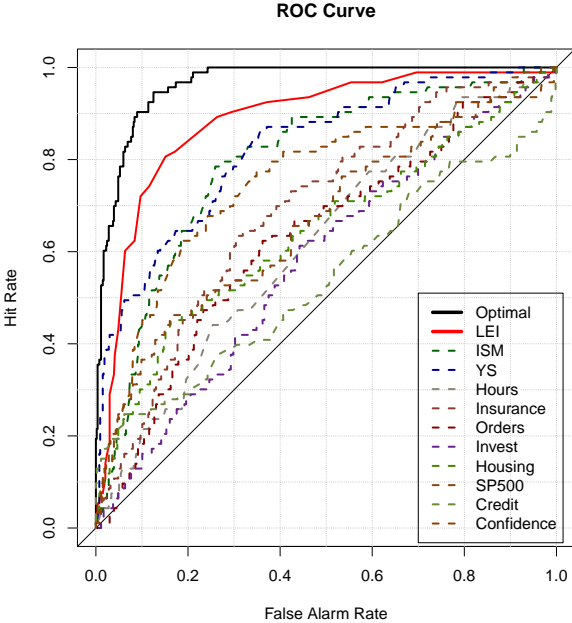
Table 1: 10-dimensional D-Vine trees and parameter estimates

pair	expansion		recession	
	family	parameter	family	parameter
$c_{1,2}$	Gaussian	0.284	270-rotated Clayton	-0.685
$c_{2,3}$	Gumbel	1.086	Independent	-
$c_{3,4}$	Survival Gumbel	1.172	Frank	2.609
$c_{4,5}$	Gaussian	0.386	Gumbel	1.269
$c_{5,6}$	Survival Gumbel	1.199	Independent	-
$c_{6,7}$	Independent	-	Independent	-
$c_{7,8}$	Independent	-	Independent	-
$c_{8,9}$	Independent	-	Independent	-
$c_{9,10}$	Independent	-	Independent	-
$c_{1,3 2}$	Frank	1.146	Gumbel	1.163
$c_{2,4 3}$	Survival Gumbel	1.084	Independent	-
$c_{3,5 4}$	Gaussian	0.121	Independent	-
$c_{4,6 5}$	Independent	-	Survival Clayton	0.393
$c_{5,7 6}$	Frank	1.584	Frank	1.510
$c_{6,8 7}$	Independent	-	Survival Clayton	0.333
$c_{7,9 8}$	Independent	-	Independent	-
$c_{8,10 9}$	Gumbel	1.077	Independent	-
$c_{1,4 2,3}$	Survival Gumbel	1.112	Independent	-
$c_{2,5 3,4}$	Independent	-	Independent	-
$c_{3,6 4,5}$	Independent	-	Independent	-
$c_{4,7 5,6}$	Gaussian	0.188	Independent	-
$c_{5,8 6,7}$	Independent	-	Independent	-
$c_{6,9 7,8}$	Independent	-	Independent	-
$c_{7,10 8,9}$	Independent	-	Independent	-
$c_{1,5 2-4}$	Clayton	0.146	Independent	-
$c_{2,6 3-5}$	Independent	-	Independent	-
$c_{3,7 4-6}$	Survival Gumbel	0.086	Independent	-
$c_{4,8 5-7}$	Independent	-	Frank	1.705
$c_{5,9 6-8}$	Independent	-	Independent	-
$c_{6,10 7-9}$	Independent	-	Independent	-
$c_{1,6 2-5}$	Independent	-	Independent	-
$c_{2,7 3-6}$	Survival Joe	1.128	Independent	-
$c_{3,8 4-7}$	Independent	-	Independent	-
$c_{4,9 5-8}$	Independent	-	Independent	-
$c_{5,10 6-9}$	Independent	-	Independent	-
$c_{1,7 2-6}$	90-rotated Gumbel	-1.147	Independent	-
$c_{2,8 3-7}$	Independent	-	Independent	-
$c_{3,9 4-8}$	Independent	-	Independent	-
$c_{4,10 5-9}$	Independent	-	Independent	-
$c_{1,8 2-7}$	Independent	-	Independent	-
$c_{2,9 3-8}$	Independent	-	Gumbel	2.120
$c_{3,10 4-9}$	Independent	-	Independent	-
$c_{1,9 2-8}$	Independent	-	Frank	-1.429
$c_{2,10 3-9}$	Independent	-	Independent	-
$c_{1,10 2-9}$	t	0.107(3.753)	Survival Clayton	0.406

Notes: $c_{i,j|k-q} \equiv c_{i,j|k,k+1,k+2,\dots,q}$ for $q > k$. For t copula, the first parameter is the correlation coefficient, while the second one in the parenthesis is the degree of freedom. The details of all other copulas are presented in Brechmann and Schepsmeier (2013).

interest spread) by exploiting the useful information contained in other predictors. However, it fails to use this additional information in an efficient manner, and the amount of efficiency loss can be represented by the non-trivial gap between the ROC curve of the current LEI with that of the optimal scheme. To summarize this difference in a single statistic, Table 2 reports the AUC for each curve in Figure 3, together with their bootstrap 95% confidence intervals. The number of replicates in the circular bootstrap is 200. Impv, the improvement of the optimal procedure over the current LEI, is 9.2%, which is significant at 5% level as its confidence interval excludes zero. Given that the AUC for LEI is already high (0.884), this improvement can be reasonably desired.

Figure 3: In-sample ROC curves



To appreciate the economic benefit from our scheme, we consider the choice of a forecaster facing two forecasting systems, namely. the optimal procedure v.s. the current LEI. Suppose the utility function of the forecaster is described by a weighted average of the hit rate and false alarm rate as $U(m, w) = mH(w) + (1 - m)(1 - F(w))$, where $m \in [0, 1]$ is the weight attached to the hit rate. Given m , the forecaster would choose w to maximize $U(m, w)$. Let $w^*(m) \equiv \operatorname{argmax}_w U(m, w)$ and $U^*(m) \equiv U(m, w^*(m))$ be the optimal solution and the maximized utility function. The forecasting system yielding the higher $U^*(m)$ is preferred for given m . For another forecaster with different m , $U^*(m)$ corresponding to the two systems

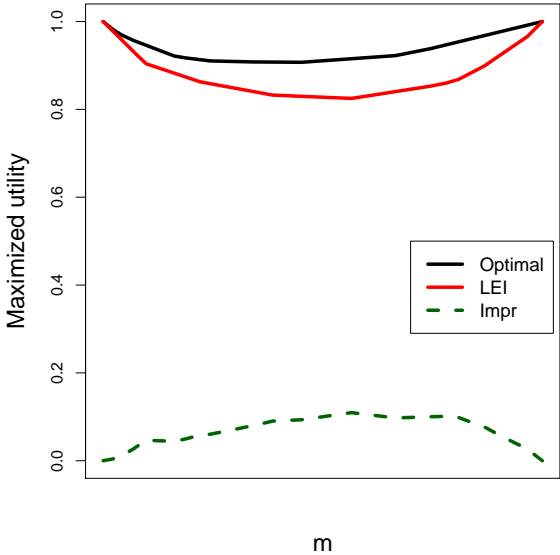
Table 2: The area under the ROC curve

Object	estimate	L	U
AUC(Optimal)	0.965	0.964	0.996
AUC(LEI)	0.884	0.811	0.940
AUC(ISM)	0.795	0.678	0.901
AUC(YS)	0.823	0.693	0.933
AUC(Hours)	0.617	0.575	0.647
AUC(Insurance)	0.690	0.642	0.746
AUC(Orders)	0.631	0.570	0.681
AUC(Invest)	0.577	0.531	0.619
AUC(Housing)	0.641	0.568	0.722
AUC(SP500)	0.668	0.588	0.719
AUC(Credit)	0.542	0.301	0.744
AUC(Confidence)	0.750	0.653	0.846
Impv	0.092	0.051	0.204

Notes: The top of the table contains the AUC value computed for each curve in Figure 3. $\text{Impv} = (\text{AUC}(\text{Optimal}) - \text{AUC}(\text{LEI})) / \text{AUC}(\text{LEI})$. Column “L” is the lower bound of the interval, while column “U” is the upper bound.

would be different as well. Figure 4 plots $U^*(m)$ for the two systems as functions of m , as well as the improvement of the optimal scheme over the current LEI in terms of the utility. Both forecasting systems are akin to each other when m is around zero or one. That is, when either $H(w)$ or $F(w)$, but not both, matters, the forecaster is indifferent between two systems. Our method performs best relative to the LEI when $m = 0.565$. The gain in utility by using our method is about 11%. Since a great amount of loss is associated with recession, this improvement can be economically significant.

Figure 4: The maximized utility as a function of the weight m

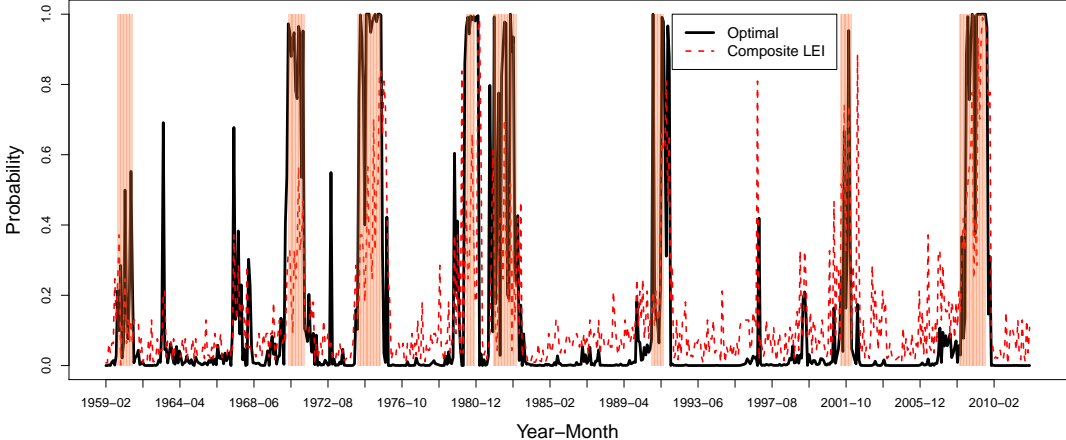


It follows from (12) that the likelihood ratio $f(X|Z = 1)/f(X|Z = 0)$ can be translated into a predictive probability of economic recession given the 10 predictors. Similarly, we can use a univariate logit model to regress Z on the LEI, and compute the fitted probabilities. Figure 5 displays the behavior of these two probabilistic series over time, along with the recessionary phases identified by NBER. It is evident that every recession since 1959 is accompanied by a higher-than-usual forecast probability for both systems. The current LEI seems to be slightly more conservative to give high probabilities during recessions. During normal periods, two episodes emerge. Prior to 1985, the optimal approach tends to generate many false signals. After 1985, its predictive probabilities always fluctuate between 0 and 0.2, with 1997 as an exception when the probabilities were as high as 0.4.⁴ Therefore, compared with the currently

⁴The predictive probabilities of the LEI are even higher during 1997.

used leading index, our procedure is more capable of identifying the incoming economic recessions throughout the sample period. Since 1985, it has also become more reliable to discriminate between the two regimes.

Figure 5: In-sample probability series



Notes: The shaded bars mark the months of economic recessions defined by NBER.

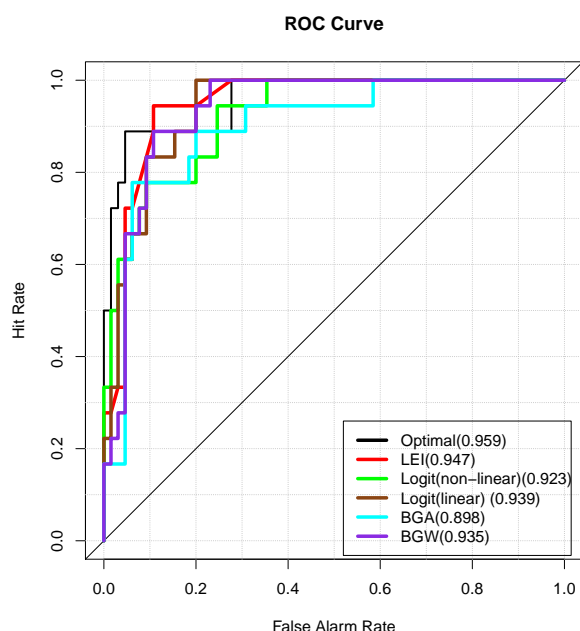
Due to the large dimension, our model might suffer from in-sample overfit. To remedy this pitfall, we intend to undertake a pseudo-real-time experiment to assess its out-of-sample performance. We are interested in whether the superiority of our procedure remains in a real time circumstance. Specifically, the whole dataset is divided into two subsamples. The first subsample (from 1959:08 to 2005:01) is used for the initial training purpose. After replacing all unknown quantities in our non-linear scheme by their sample counterparts, we evaluate our forecasting rule at the values of the 10 predictors next month to generate a six month ahead forecast. That is, we attempt to predict whether a recession would occur in 2005:02 using the information contained in the 10 predictors in 2004:07. The only difference from the previous in-sample analysis is that all unknown quantities in the forecasting rule are estimated based on the observations from 1959:08 to 2005:01. We can repeat the same computation by adding one more observation to the training sample and generate a new six month ahead forecast. That is, we map the 10 predictors in 2004:08 to a forecast of recession in 2005:03 using the forecasting rule estimated using the subsample from 1959:08 to 2005:02. This rolling-window procedure can be iterated until the end of the sample, yielding 84 out-of-sample pairs of forecasts and actuals. Note that the target we are seeking to predict is not used for

estimation, so that it is uncertain that our method is still better than the current LEI.

Figure 6 presents the ROC curves for the pseudo-real-time experiment. As expected, the current LEI improves in this study with its AUC rising from 0.884 to 0.947. A possible reason is that one of the predictors, namely, the Leading Credit Index constructed by Levanon *et al.* (2014), was found to lead the economic recession driven by financial turmoil very well. Given that the 2007-2009 recessions are closely linked with the financial crises following the collapse of Lehman Brothers, it is unsurprising that the current LEI performs better during this particular period. When compared with our procedure, the LEI does not entail many parameters to estimate and thus it is likely to be stable out-of-sample. This agrees with the argument in Timmermann (2006)—the simple average combination scheme may beat the complex data-driven method. As an illustration, we fit an expanded version of a logit regression with the 10 predictors, their squared terms and interaction terms being added as the explanatory variables. As mentioned in Section 2, this logit regression is a special case of our optimal scheme when each of the 10 predictors follows normal distribution and all bivariate copulas are Gaussian. In a separate in-sample study, we found that this logit regression, as a non-linear combination scheme, performs equally well with the procedure based on the D-Vine trees. However, it loses out-of-sample predictive precision in Figure 6 with the AUC being 0.923. Besides normality, we can also restrict the two covariance matrices to be equal, which is equivalent to a conventional logit model with the linear index function. Once the higher-order terms are dropped, the logit model gets better with the out-of-sample AUC being 0.939. This implies that the striking in-sample performance of the complex logit model is mainly attributed to overfit as it has 66 unknown coefficients to estimate.

Figure 6 also shows the two ROC curves generated by the linear combination method proposed by Bates and Granger (1969). To apply this method, we first map each of the 10 predictors into a predictive probability using a univariate logit regression. The first ROC curve (named “BGA”) is produced by using a simple average of these 10 probabilities, while the second one (named “BGW”) is calculated from a weighted average and the weights are determined from an auxiliary regression, as suggested by Granger and Ramanathan (1984). Lahiri and Yang (2015) compared these linear schemes with the optimal procedure in terms of calibration and resolution and they concluded that they are lack of calibration and are

Figure 6: Out-of-sample ROC curves



Notes: In the legend box, the numbers in the parenthesis are the values of AUC.

generally not optimal to maximize ROC curve. This point is also confirmed in Figure 6 in that their AUC values are 0.898 (for “BGA”) and 0.935 (for “BGW”), both of which are even lower than that of the LEI. Despite the large number of parameters to estimate, our optimal procedure works remarkably well. Among all combination schemes, it is still the best one with the AUC being 0.959. Lahiri and Yang (2015) conducted a counterfactual experiment to examine what can explain the better performance of our non-linear scheme relative to the linear benchmark. In general, if the two conditional distributions of predictors are more distinct from each other, a higher improvement can be made by using the non-linear scheme. Given the different distributions and dependence patterns across regimes in Figure 2, our method is warranted.

4 Concluding remarks

This paper makes a multivariate extension of the non-linear forecast combination scheme developed by Lahiri and Yang (2015). We provide a convenient, computational-affordable

method to combine multiple predictors. Unlike many previously proposed alternatives, the resulting combined forecasts are optimal to maximize the discriminatory power as measured by the ROC curve. In order to implement this scheme, one has to estimate the conditional distributions of predictors given the binary target. When the number of predictors are larger than two, vine-copula facilitates the modeling of the marginal distribution of each predictor and their dependence structure separately. A two-step procedure is advocated in practice. In the first step, we estimate the density of each predictor by the standard non-parametric method. The copula dependence parameters are estimated sequentially in the second step by plugging in the first-step estimates. To account for the sampling variability, a circular block bootstrap is used to construct the confidence interval of the area under the ROC curve. The relevance of our approach is demonstrated in an empirical illustration, where we combine the 10 predictors of the Leading Economic Index of The Conference Board to construct a new leading index for U.S. business cycles. Compared with the currently used LEI, the predictive probabilities implied by our model tend to distinguish the underlying economic states significantly better both in- and out-of-sample. In contrast to some popular competitors, its real-time performance is pretty robust even with a large number of unknown quantities to estimate.

Although a D-Vine structure with an appropriate ordering is always used in our application, we do not claim this particular specification is universally applicable. Other structures, such as a C-Vine, may also provide satisfactory fits. With many predictors at hand, numerous vine structures are available to choose from. Given a sample, specifying a different structure may lead to different estimates, which in turn imply different relationship among predictors. To avoid this arbitrariness in specification, a model selection criterion or test is necessary. Despite a large number of papers dealing with copula specification, to the best of our knowledge, there is no prior research on vine copula providing a good procedure to choose the structure. Without a Monte Carlo simulation, it is unclear if the misspecification of the vine structure and pairwise copulas would result in a sizeable bias for the estimated ROC curve. Further, in our empirical illustration, we only consider six month ahead forecasts. However, no guideline is provided as to which horizon is suitable for the use of the current LEI. In light of this observation, the relative poorer performance of the LEI might be the result of select-

ing an incorrect horizon. Thus, a more extensive study is called for to investigate whether our method is uniformly better across horizons and, if it is not, at what horizons it is better. Finally, as shown in Figure 5, the performance of our scheme is not necessarily good at any time. Before the mid of 1980s, it falsely gave a couple of recession signals, some of which are as high as 0.7. One possible explanation is that some fundamental structures within the U.S. economy might have changed around the mid of 1980s so that the mechanism in which some predictors lead the business cycles may change as well. Levanon *et al.* (2014) discussed some structural breaks occurring during this period, including the shift of the target of monetary policy, which may change the pathway the financial sector impacts the real economy. In fact, these changes are responsible for the replacement of the real M2 by the Leading Credit Index as one component of the LEI in 2001 because the later proves to work better since 1990s. We have split the sample into two periods using 1986:01 as the cutoff and we estimated the model separately using each subsample. As expected, the parameter estimates and the ROC curves change drastically across samples, indicating the existence of the break.⁵ As a solution, we can discard the first period and only use the sample after 1986:01 to estimate our model and construct the forecast. This makes sense if we assume no breaks would occur. However, if breaks have occurred in the past, surely they are also likely to happen in the future. To make our forecast remain optimal in the presence of structural change, we have to take the uncertainty of future breaks into account in specification, estimation and forecast. To this end, the recent work by Chen and Niu (2014), Pesaran et al. (2006), and Pesaran et al. (2013) can be integrated into our framework. These interesting topics remain for future investigation.

⁵Results are available upon request.

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