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Firms are challenged to improve the effectiveness of cross-selling campaigns. The authors propose a customer-response model that recognizes the evolvement of customer demand for various products; the possible multifaceted roles of cross-selling solicitations for promotion, advertising, and education; and customer heterogeneous preference for communication channels. They formulate cross-selling campaigns as solutions to a stochastic dynamic programming problem in which the firm's goal is to maximize the long-term profit of its existing customers while taking into account the development of customer demand over time and the multistage role of cross-selling promotion. The model yields optimal cross-selling strategies for how to introduce the right product to the right customer at the right time using the right communication channel. Applying the model to panel data with cross-selling solicitations provided by a national bank, the authors demonstrate that households have different preferences and responsiveness to cross-selling solicitations. In addition to generating immediate sales, cross-selling solicitations also help households move faster along the financial continuum (educational role) and build up goodwill (advertising role). A decomposition analysis shows that the educational effect (83%) largely dominates the advertising effect (15%) and instantaneous promotional effect (2%). The cross-selling solicitations resulting from the proposed framework are more customized and dynamic and improve immediate response rate by 56%, long-term response rate by 149%, and long-term profit by 177%.

Keywords: cross-selling campaign, customer relationship management, customer-centric marketing, dynamic programming, analytical decision-support tools, hidden Markov model, evolving customer response, multichannel communication

Cross-Selling the Right Product to the Right Customer at the Right Time

Cross-selling is the practice of selling an additional product or service to an existing customer. It ranks as a top strategic priority for many industries, including financial services, insurance, health care, accounting, telecommunications, airlines, and retailing. Despite the increasing investment in cross-selling programs, firms have found that these million-dollar marketing campaigns are not profitable (Authers 1998; Business Wire 2000; Rosen 2004). The average response rate as measured by a customer purchase within three months after a cross-selling campaign is approximately 2% (Business Wire 2000; Smith 2006). A managerial challenge is to improve the response rates of a cross-selling campaign while avoiding the targeting of customers with irrelevant messages.

Most current cross-selling campaigns are designed with a "Let's find the customers who are most likely to respond" orientation. Firms begin cross-selling campaigns by setting a time schedule (e.g., mail the promotional material in one month) and then selecting a communication channel (e.g.,

phone, e-mail, mail) for this campaign. Analysts then develop a customer-response model with the purchase decision as a dependent variable and product ownership and customer demographics as explanatory variables. Finally, after estimating the customer-response model, analysts compute the expected profit, and firms schedule all customers with positive expected profits to receive the promotion. If the firm must heed a budget constraint, it will only solicit the most profitable customers. We refer to this process as "campaign-oriented cross-selling."

We argue that an improved customer-centric orientation for cross-selling is, "How do we introduce the right product to the right customer at the right time using the right communication channel to ensure long-term success?" Conceptually, customer demand for financial services depends on the customer's evolving financial maturity (Kamakura, Ramaswami, and Srivastava 1991; Li, Sun, and Wilcox 2005). Thus, each individual customer's preferences and responsiveness to cross-selling solicitations may change over time, and the marketer must track and anticipate these changes (Netzer, Lattin, and Srinivasan 2008). In addition, cross-selling solicitations may provide more than just a promotional incentive that immediately stimulates purchase. Cross-selling can create enduring relationships between a customer and the firm by serving as a general advertisement for the brand and a signal of quality and to educate consumers about the scope of product offerings and how various products meet their long-term financial needs. Ultimately, this requires the marketer to have a long-term view and generate dynamic solicitations in accordance with the customer's evolving financial status and preferences to maximize the long-term financial payoff (Sun, Li, and Zhou 2006).

The focus of our research is to take up this challenge and understand the many roles of solicitations in a cross-selling campaign, how it interacts with customer purchase decisions, and how cross-selling can be improved. More specifically, we address the following open research questions: How do cross-selling solicitations interact with customer decision processes about purchases of financial products? Do cross-selling solicitations have long-term effects in addition to generating immediate purchase? If so, how can we decompose the short- and long-term effectiveness of cross-selling campaigns? Do customers differ in their preference for communication channels? How should a firm best use the long-term role of cross-selling solicitations when making cross-selling solicitation decisions?

We develop a multivariate customer-response model with hidden Markov transition states to statistically capture the possibility that customer demand for various financial products is governed by evolving latent financial states, during which customers have different preference priorities and responsiveness to cross-selling solicitations for various financial products. We capture long-term effects of solicitations by allowing cross-selling to change the speed of customer movement along the financial maturity continuum. Across-customer heterogeneity is captured through a hierarchical Bayesian approach. We calibrate our model to customer purchase histories provided by a national bank.

Using the estimated customer-response parameters, we formulate the bank's cross-selling decisions as solutions to a stochastic dynamic programming problem that maximizes customer long-term profit contribution. This proposed dynamic optimization framework enables us to integrate intracustomer heterogeneity (the evolving financial states of each customer) and long-term dynamic effects of cross-selling solicitations. It results in a sequence of solicitations that represent an integrated multistep, multisegment, and multichannel cross-selling campaign process to optimize the choice and timing of these messages. We compare our results with current industry practice and several alternative cross-selling approaches that ignore intracustomer heterogeneity, disregard the cumulative effects of cross-selling, and make cross-selling decisions myopically. Compared with current practice observed in our data set, our proposed approach improves immediate response rate by 56%, long-term response rate by 149%, and long-term profit by 177%.

CROSS-SELLING LITERATURE

We summarize previous academic research on cross-selling and customer lifetime value (CLV) analysis in Table 1. Existing literature focuses on developing methods to more accurately predict purchase probabilities for the next product to be purchased and is useful in supporting campaign-centric cross-selling or the next product to be cross-sold. With the exception of Kumar et al. (2008), none of the existing cross-selling studies use information on cross-selling solicitations, and little is known about how cross-selling solicitations affect customer purchase decisions in the long run. Customer lifetime value in campaign-oriented cross-selling is usually treated as another segmentation variable to differentiate profitable customers from unprofitable ones. However, Rust and Chung (2006) and Rust and Verhoef (2005) point out the problem with this approach: The bank's intervention changes a customer's future purchase probabilities.

Our study contributes to the existing literature on cross-selling in the following ways. First, we directly observe the cross-selling solicitations (or promotions) made to customers in our empirical study. Thus, this is the first study to explicitly model how customers dynamically react to cross-selling solicitations and measures the effectiveness of cross-selling solicitations in the short and long run. Second, we relax the strong assumption that customer responsiveness to solicitations is fixed over time and allow the responsiveness to solicitations to change over time. The evolving state structure enables us to investigate how effectiveness of solicitations cross-selling different products varies with customer financial states or communication channels. Third, we recognize and model the long-term effects of solicitation in the customer response model (which we refer to as the educational and advertising roles). These effects have been documented by industry reports (Strazewski 2010) but not in the academic literature. Fourth and most important, we demonstrate that intracustomer heterogeneity and long-term effects of solicitations require the firm to take a long-term view and adopt a dynamic programming approach when making solicitation decisions.

DATA DESCRIPTION

A national bank that offers a complete line of retail banking services provided our data. The data set consists of monthly account opening and transaction histories, cross-selling solicitations about the type of product promoted and the communication channels used (i.e., e-mail or postal

Table 1

SUMMARY OF LITERATURE ON CROSS-SELLING AND CLV ANALYSIS

Study	Product Ownership	Measuring Effectiveness of Cross-Selling	Long-Term Role of Cross-Selling	Intracustomer Heterogeneity	CLV	Dynamic Programming
Edwards and Allenby (2003)	✓					
Kamakura et al. (2003)						
Knott, Hayes, and Neslin (2002)						
Kamakura, Ramaswami, and Srivastava (1991)						
Li, Sun, and Wilcox (2005)						
Berger and Nasr (1998)	✓				✓	
Dwyer (1989)						
Fader, Hardie, and Lee (2005)						
Jackson (1996)						
Mulhern (1999)						
Rust, Lemon, and Zeithaml (2004)						
Lewis (2005a, b)	✓		✓		✓	✓
Rust and Chung (2006)						
Reinartz, Thomas, and Kumar (2005)	✓	✓			✓	
Rust and Verhoef (2005)						
Venkatesan and Kumar (2004)						
Kumar, Venkatesan, and Reinartz (2008)						
Venkatesan, Kumar, and Bohling (2007)						
Du and Kamakura (2006)	✓		✓	✓		
Netzer, Lattin, and Srinivasan (2008)						
Our study	✓	✓	✓	✓	✓	✓

mail), and demographic information (compiled by a marketing research firm to which the bank subscribes) of a randomly selected sample of 4000 households for 15 financial product groups during a total of 27 months from November 2003 to January 2006.

We grouped the 15 products into seven categories: checking, savings, credit cards, lending, certificates of deposit (CDs), investment, and others.[1] Therefore, our purchase variable records when a specific account is opened. Because there are multiple financial products within a category, repeat purchases are recorded as a purchase of a financial product (category). For example, a customer with an existing free checking account opens a second interest checking account. This is represented in our data as a purchase. In addition, our analysis is at the household level, which may be made up of many people. Repeat purchases of similar products can be purchased by or for other household members. In short, we do not distinguish new products within a category, repeat purchases by the same person, or new purchases by other household members. Third, it is rare that

customers make more than one purchase in a category within a single month, so we focus on an indicator of purchase within the category and not the number of items purchased.

Our calibration sample consists of 2000 randomly selected households that received a total of 12,590 solicitations and made a total of 4948 purchases during the 27 months. We have a cross-sectional validation sample with another 2000 randomly selected households that were contacted 12,797 times and made 5038 purchases during the same 27 months. In addition, for cross-time validation, we used the first 26 months of these 4000 households for estimation and retained the final month for a holdout sample.

Table 2 gives a brief description of the variables this study uses for the whole sample. The households have average total assets of $97,243.40 as estimated by a marketing research company. The variable COMP measures the share of wallet, or percentage of customer assets that are allocated to other financial institutions. This variable is just the marketing research company's estimate and is a static measure of competition from other financial institutions. We observed that the number of solicitations sent to the average household during the 27 months is 6.35. The bank deliberately aims to avoid overwhelming its customers with solicitations and limits its marketing activities to approximately one solicitation per quarter. The bank provided the profit information for each household and every account, calculated using full absorption accounting based on the customer's usage of the bank's services. The average profit margin per account per month is $14.71. We also learn from bank managers that the average cross-selling solicitation costs approximately $.50 and $.05 per message for postal and e-mail, respectively.

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[1]Checking includes various types of checking accounts; savings includes money market and savings accounts; credit cards include credit cards and bank cards; lending includes mortgage, term loans, and secure credit line; CDs include time deposits or CDs; investments include annuity, trusts, and security investments; and other includes safe deposit box and other services. This classification follows the practice of the bank and helps us avoid estimation issues related to data scarcity. We acknowledge that this is a simplification, but it is an accepted practice (Edwards and Allenby 2003; Kamakura, Ramaswami, and Srivastava 1991; Li, Sun and Wilcox 2005), and we believe it preserves the basic structure of the problem. The exercise of aggregating across both similar products and household members is related to the data we are provided with. However, the proposed model can be applied to data without data aggregation.

Table 2

SAMPLE STATISTICS

Variables	M or Frequency	SD	Minimum	Maximum
Purchase Transactions				
Checking	.008	.091	0	1
Savings	.007	.084	0	1
Credit cards	.002	.045	0	1
Lending	.003	.055	0	1
CDs	.003	.054	0	1
Investment	.001	.036	0	1
Others	.007	.081	0	1
Frequency of Solicitations				
Checking	.001	.002	0	1
Saving	.004	.068	0	2
Credit cards	.009	.094	0	2
Lending	.019	.138	0	2
CDs	.001	.003	0	1
Investment	.008	.092	0	3
Others	.040	.210	0	4
Frequency of All Solicitation				
Mail	.069	.26	0	3
E-mail	.010	.103	0	2
Demographics				
Percentage of assets outside the bank	.77	.67	0	1
Tenure with the bank	66.46	97.65	0	1260
Number of transactions	12.28	25.95	0	468
Age	51.49	14.54	18	98
Gender (male)	.59	0.49	0	1
Household size	2.42	1.17	1	8
Income[a]	5.49	2.30	1	9
Profit				
Average account profit	14.71	230.86	−36,416.4	34,081.3
Average account balance	17,103.6	81,811.9	604.9	5,607,220
Checking	1676.4	14,358.5	−4222.8	2,757,603
Savings	3234.2	22,838.9	−3162.4	1,309,694
Credit cards	267.2	3265.7	−25,000	258,748.6
Lending	4144.5	36,515.9	−37,580.4	2,222,000
CDs	1927.4	16,675.8	−10,769.8	619,833.9
Investment	4575.4	55,023.6	0	4,796,926
Others	1274.1	39,386.6	−332.8	5,607,224
TACCT	5.19	4.87	1	155
NACCT	3.59	3.87	0	92
Assets	97,243.4	187,748.1	0	2,000,000
Mean of balance change in $1,000s	−6.35	.19	−.033	3.8
Variance of balance change in $1,000s	7.36	.5	0	13.8
Cumulative number of accounts closed	1.59	2.27	0	62

[a]Income is reported as an ordinal variable that ranges takes on the values: 1 (under $15k), 2 [$15K, $20K), 3 [$20K, $30K), 4 [$30K, $40K), 5 [$40K, $50K), 6 [$50K, $75K), 7 [$75K, $100K), 8 [$100K, $125K), and 9 [$125K and above). Thus, the average of 5.5 implies an average income above $50K.

CUSTOMER-RESPONSE MODEL

We observed the set of financial products and services a household purchased and the cross-selling campaign messages it received each month. The bank needed to evaluate how the cross-selling solicitations interact with customer decision process, determine the short-and long-term consequences of these campaign messages on household cross-buying decisions, and predict when customers will open a new account. The core of our model is a multivariate probit model that predicts whether a household will decide to open a new account in a given month. The covariates within the probit model reflect how the customer's decisions are influenced by cross-selling efforts of the bank, as well as the household's characteristics. The parameters of this probit model depend on a latent financial state for each customer that we estimate. This latent state is time dependent, and its dynamics explain how a customer's financial status can change and influence a customer's response to marketing efforts. The hierarchical specification of our model relates the probit parameters to a household's characteristics. To optimize consumer response to cross-selling efforts, we first specify the long-term profit for a customer and then show the integrated multistep, multisegment, and multichannel cross-selling campaign solutions.

A Multivariate Probit Model of Purchase

We use an indicator variable Y_{ijt} to represent a household's purchase decisions:

$$(1) \quad Y_{ijt} = \begin{cases} 1 & \text{if product j is purchased by household i at time t} \\ 0 & \text{otherwise} \end{cases},$$

where subscript i represents the household ($i = 1, \ldots, I$), j represents the product category ($j = 1, \ldots, J$), and t represents

the month $(t = 1,\dots, T)$.[2] We index the household's latent financial state using s, which we explain subsequently.

As the cross-selling literature shows, factors such as promotion or solicitation, the bank's efforts to maintain the relationship with the customer, available financial resources, the cost of switching to another financial institution, income, and the competition are likely to determine a household's decisions regarding the purchase of financial products (Kamakura, Ramaswami, and Srivastava 1991; Li, Sun, and Wilcox 2005). Accordingly, we assume that whether customer i purchases financial product j at time t can be explained by the following latent utility function:

$$(2) \quad U_{ijt}(s) = \beta_{0ij}(s) + \sum_{k=1}^{K}[\beta_{1ij}(s) + \beta_{2ik}(s)]Z_{ijkt}$$

$$+ \beta_{3i}(s)\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{\tau=1}^{t-1}Z_{ijk\tau} + \beta_{4i}(s)\mu_{\Delta BAL_{i(t-1)}}$$

$$+ \beta_{5i}(s)\sigma_{\Delta BAL_{i(t-1)}} + \beta_{6i}(s)NACCT_{ijt}$$

$$+ \beta_{7i}(s)TRANS_{i(t-1)} + \beta_{8i}(s)TENURE_{it}$$

$$+ \beta_{9i}(s)COMP_{it} + \beta_{10i}(s)INCOME_{it} + \epsilon_{ijt}(s)$$

for all $i = 1, \dots, I$, $j = 1, \dots, J$, and $t = 1, \dots, T$. Here, $\beta_{0ij}(s)$ captures household i's intrinsic preference for purchasing product j in state s. The following sections briefly describe each of our variables.

Instantaneous promotional effects of solicitations. The variable Z_{ijkt} is the number of solicitation messages household i receives for product j using channel k during month t, where k = 1 is postal mail and k = 2 is e-mail. Its product-specific coefficient $\beta_{1ij}(s)$ measures the immediate impact of promotional effects from a cross-selling solicitation of product j on the household's purchase probability of product j. For brevity, we refer to this as the instantaneous promotional effect of cross-selling solicitations, which we expect a priori to positively impact product purchase. These coefficients are the ones that most analysts of cross-selling campaigns rely on to measure the (immediate) effectiveness of their campaigns. To take into account channel differences, we also include $\beta_{2ik}(s)$, which measures the differential instantaneous effect of the message being sent through channel k. Comparing $\beta_{1ik}(s)$ across products and $\beta_{2ik}(s)$ across communication channels reveals how the immediate

effects of cross-selling campaigns differ across financial products and communication channels, respectively.

Advertising effect of solicitations. The cumulative number of cross-selling solicitations household i receives through period $t - 1$ is $\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{\tau=1}^{t-1}Z_{ijk\tau}$. This variable measures the bank's total outreach efforts. It is included to measure the possibility that households interpret the cumulative impression of the bank's cross-selling effort as its good intention to maintain a relationship with the customer or as a signal of the bank's quality. We label this long-term, accumulative influence as the advertising effect of cross-selling campaigns (Little 1979; Lodish et al. 1995).

Account transactions. The terms $\mu_{\Delta BAL_{it-1}}$ and $\sigma^2_{\Delta BAL_{i(t-1)}}$ are the mean and variance, respectively, of the change in balances we observe through time $t - 1$. Their coefficients $\beta_{4ij}(s)$ and $\beta_{5ij}(s)$ measure the effects of change of these two variables on the purchase probability. The variable $NACCT_{ijt}$ is the number of accounts in all other product categories except j owned by household i up to time t. We include this to control for the possibility that the currently owned accounts in other product categories may compete for financial resources and thus affect the probability of purchasing a new financial product j. We expect the coefficient $\beta_{5ij}(s)$ to be negative. Finally, $TRANS_{i(t-1)}$ is the total number of transactions the household conducted at the bank by the end of time $t - 1$, which represents a sign of the quality of the customer–seller relationship (Anderson and Weitz 1992; Kalwani and Narayandas 1995; Reinartz, Thomas, and Kumar 2005).

Household characteristics. We use $TENURE_{it}$ to refer to the number of years since the household opened its first account at the bank. It approximates customer inertia to switch to another financial institute. We define $COMP_{it}$ as the percentage of assets not allocated to this bank, which approximates possible competition, and $INCOME_{it}$ is an ordinal measure of the household income for time t. These three variables control for switching cost, competition, and income effects.

Stochastic error structure. We use $\epsilon_{ijt}(s)$ to define the unobservable random shock that determines the purchase of product j in state s at time t. We let vector $\epsilon_{it}(s)$ represent the J random shocks and assume the unobserved part of the J utilities are correlated:

$$(3) \quad \epsilon_{it}(s) \sim MVN[0, \Sigma], \epsilon_{it}(s) = [\epsilon_{i1t}(s), \epsilon_{i2t}(s), \dots, \epsilon_{iJt}(s)]'.$$

Given the error structure we impose on Equation 3, our model is a canonical multivariate probit model specification, and thus the probability of the observed vector of product purchases for household i at time t in state s is given by

$$(4) \quad Prob(\mathbf{Y}_{it}|s) =$$

$$\int_{M_j}(2\pi)^{-J/2}|\Sigma|^{-1/2}\exp\left\{-\frac{1}{2}\epsilon'_{it}(s)\Sigma^{-1}\epsilon_{it}(s)\right\}d\epsilon_{it}(s),$$

where $M_j = (-\infty, 0)$ if $Y_{ijt} = 0$ and $(0, \infty)$ if otherwise. Finally, $\mathbf{Y}_{it}$ is the observed profile ($J \times 1$ vector) of binary choices of product j of household i at time t.

A Household's Financial State

The parameters of our multivariate probit model (Equation 2) are indexed by state s at each time. This state cap-

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[2]Treating opening of an account as the dependent variable follows the cross-selling literature and industry practice. Most of the cross-selling campaigns solicitations are sent to customers with the goal of informing them about the existence of this product. Existing studies on cross-selling (Edwards and Allenby 2003; Kamakura, Ramaswami, and Srivastava 1991; Knott, Hayes, and Neslin 2002; Li, Sun, and Wilcox 2005) all define opening of account to measure the effectiveness of cross-selling solicitations. In our data, 97% of these promotions are cross-selling for products that the customer does not own. Most of the solicitations are about the availability and benefit of the cross-sold product and are not price related. Thus, we measure the effectiveness of cross-selling campaigns by responsiveness to open new accounts. We calculated correlations between solicitation and purchase and between solicitation and balance, and they indicate that the correlation is weak between solicitation and balance, while the correlations are strong and significant between solicitation and opening of accounts. We estimated a simultaneous equations model with balance as the dependent variables, and the impact of cross-selling solicitations is insignificant.

tures a consumer's latent financial maturity, which may govern a household's sequential demand for various financial products (Kamakura, Ramaswami, and Srivastava 1991; Li, Sun, and Wilcox 2005). Our states are consistent with the buyer–seller relationship theories developed by Aaker, Fournier, and Brasel (2004), Dwyer, Schurr, and Oh (1987), and Fournier (1998). This research suggests that relationships evolve through several discrete phases as a result of changes in the environment and interactions between the partners. The transitions between relationship stages may be triggered by discrete encounters such as transactions and the firm's marketing contacts between relationship parties.

On the basis of these theories, we propose a probabilistic model that allows households to have different intrinsic preferences for financial products and heterogeneous responsiveness to cross-selling efforts in each latent state. We assume a household can be allocated to one of S latent states at each time. The transition among these states is governed by a first-order continuous-time discrete-state hidden Markov model (HMM) (Li, Liechty, and Montgomery 2002; Montgomery et al. 2004). Moon, Kamakura, and Ledolter (2007), Du and Kamakura (2006), and Netzer, Lattin, and Srinivasan (2008) employ a similar discrete-time HMM to investigate competitive promotions, customers' unobserved life stages, and relationship states, respectively.

For brevity, we interpret our latent states as an indicator of the household's financial state. However, we acknowledge that our states may not solely reflect a consumer's financial status. Rather, these states could reflect an amalgamation of the customer's financial well-being, knowledge and experience with financial products, customer life stage, and customer relationship with the bank. Our interpretation of states is based on a comparison of the estimated coefficients different across states and summary statistics. However, our interpretation and labeling of financial states are not unique, just as a label for a segment in cluster analysis or factor in factor analysis is not unique.

An HMM of Financial States[3]

We use an $S \times S$ matrix M_{it} to denote the probabilities for household i to transition to another state at time t:

$$(5) \qquad M_{it} = \begin{bmatrix} 0 & P_{it12} & \cdots & P_{it1S} \\ P_{it21} & 0 & \cdots & P_{it2S} \\ \vdots & \vdots & \ddots & \vdots \\ P_{itS1} & P_{itS2} & \cdots & 0 \end{bmatrix}.$$

Each element in the transition matrix P_{itmn} represents household i's probability of transitioning from state m at $t-1$ to state n at time t. Therefore, $0 \leq P_{itmn} \leq 1$, and the row sum is 1.

The diagonal elements of M_{it} are zeroes, because we do not allow same-state transitions. Instead, we capture persistence within a state as a waiting time for the state, which is the duration a household stays in one particular state. We

define $W_{it}(s)$ as the waiting time in state s and assume it follows a gamma distribution in a continuous time domain (Montgomery et al. 2004):

$$(6) \ \Pr[W_{it}(s) \mid \lambda_{it}(s), k_i(s)] = \frac{k_i(s)^{\lambda_{it}(s)}}{\Gamma[\lambda_{it}(s)]} W_{it}(s)^{\lambda_{it}(s)-1} e^{-W_{it}(s)k_i(s)},$$

where $\lambda_{it}(s)$ is the shape parameter and $k_i(s)$ is the inverse scale parameter for state s. Note that if $\lambda_{it}(s) = 1$, we have an exponential distribution. Being household specific, $\lambda_{it}(s)$ and $k_i(s)$ determine how long a household i stays in state s. More specifically, the expected waiting time until the next state equals the ratio of the shape parameter to the inverse scale parameter:

$$(7) \qquad E[W_{it}(s)] = \frac{\lambda_{it}(s)}{k_i(s)}.$$

Unlike the homogeneous HMM Du and Kamakura (2006), Montgomery et al. (2004), and Moon, Kamakura, and Ledolter (2007) use, we adopt a heterogeneous HMM and allow the household's waiting time (e.g., the shape parameter $\lambda_{it}(s)$ in Equation 6) to be affected by the household's total prior experience with the financial products and the intensity of cross-selling efforts. Specifically, we assume $\lambda_{it}(s)$ follows a log-normal distribution:

$$(8) \qquad \log[\lambda_{it}(s)] \sim N[\bar{\lambda}_{it}(s), \sigma_{\lambda}^2],$$

where its mean $\bar{\lambda}_{it}(s)$ is a function of the household's total experience with financial products and the intensity of cross-selling campaigns:

$$(9) \ \bar{\lambda}_{it}(s) = \alpha_{0i}(s) + \alpha_{1i}(s)TACCT_{i(t-1)} + \sum_{j=1}^{J} \alpha_{2ij}(s) \sum_{k=1}^{K} Z_{ijkt}$$

$$+ \sum_{k=1}^{K} \alpha_{3ik}(s) \sum_{j=1}^{J} \sum_{t_s=1}^{t-1} Z_{ijkt_s}$$

$$+ \sum_{k=1}^{K} \alpha_{4ik}(s) \left(\sum_{j=1}^{J} \sum_{t_s=1}^{t-1} Z_{ijkt_s} \right)^2 + \alpha_{5i}(s)COMP_{it}$$

$$+ \alpha_{6i}(s)INCOME_{it} + \alpha_{7i}(s)CLOSE_{it-1}.$$

The coefficient $\alpha_{0i}(s)$ captures a household's intrinsic tendency to stay in state s.

Past purchases. Variable $TACCT_{i(t-1)}$ denotes the total number of financial product categories household i owns up to time $t-1$. This variable approximates the household's total experience with financial products and thus its financial knowledge, and its coefficient $\alpha_{1i}(s)$ measures how knowledge regarding financial products affects the waiting time in state s.

Educational role of solicitations. The variable $\Sigma_{k=1}^{K} Z_{ijkt}$ measures the cumulative number of solicitations across all channels that household i receives at time t on product j, and its coefficient $\alpha_{2ij}(s)$ measures whether receiving solicitations on product j at time t changes the length of time a household stays in the same state. If this coefficient is negative, it implies more solicitations for product j will lessen the time in state s. The instantaneous promotional effect of solicitations from Equation 2 contemporaneously and

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[3]The proposed HMM is a continuous-time Markov model. An alternative is the discrete-time Markov model with nonzero diagonal elements in the state transition matrix. We ran this alternative model and obtained very similar results.

directly affects a household's decision to purchase a product. However, the effect of solicitations as measured by $\alpha_{2ij}(s)$ is indirect because it may help move households to states in which they are more receptive to future cross-selling efforts. We label this indirect effect the educational role of cross-selling. In addition, note that $\alpha_{2ij}(s)$ is product specific. Comparing these coefficients across the products (j) shows the varying effectiveness of educational roles of solicitation cross-selling these products in each state.

Our use of the term "education" is meant to convey the sense that solicitations help inform customers about the depth, variety, and benefit of product offers that can meet their future financial needs. Given the complexity of financial products, banks must provide information to inform their customers. Therefore, we hypothesize that these messages have an educational effect on the consumer's readiness to purchase financial products. The educational role of cross-selling is similar to the informative role of advertising (Mehta, Chen, and Narasimhan 2008; Narayanan and Manchanda 2009), which is meant to raise awareness or knowledge of a product. However, we caution the reader that the "educational" label is speculative on our part, because we cannot explicitly measure an increase in consumers' knowledge from cross-selling messages.

Cumulative effect of solicitations. The variable $\Sigma_{j=1}^{J}\Sigma_{t_s=1}^{t-1}$ Z_{ijkt_s} measures the total number of solicitations for a particular channel k across all J product lines that household i receives up to time $t-1$ since the beginning of its current state s (t_s represents the time index when state s starts), and $\alpha_{3ik}(s)$ is a channel-specific coefficient that captures whether the educational role (if it exists) differs across communication channels. We also include its squared term to capture the possible diminishing effectiveness of the educational role when a household receives too many solicitations through channel k as in Venkatesan and Kumar (2004) and Venkatesan, Kumar, and Bohling (2007).

Household characteristics. Including $COMP_{it}$ and $INCOME_{it}$ captures how the external factors influence a household's waiting time in state s. The variable $CLOSE_{it-1}$ is the cumulative number of accounts closed up to the end of the last period. Including this variable allows us to take into account the possibility that some households may gradually close their accounts before leaving the bank. The coefficient $a_{7i}(s)$ captures the impact of account closing on the waiting time.

Initial Financial State Probabilities of HMM

We define the initial state probabilities of household i residing in state s for s = 1, ..., S at time 0 as a vector $\Pi_i = [\pi_i(1), ..., \pi_i(S)]'$. The row vectors of the transition matrix and the vector of initial starting probabilities are assumed to follow a Dirichlet distribution:

$$(10) \qquad \mathbf{P}_{itj} \sim D(\tau_{itj}), \; \Pi_i \sim D(\eta_{is}),$$

where $\mathbf{P}_{itj}$ denotes the jth row of the transition matrix $\mathbf{P}_{it}$, and τ_{itj} and η_{is} refer to the hyperparameters for the transition and starting probabilities, respectively. Similar to the specification of the waiting time intensity, we assume τ_{itj} and η_{is} follow a log-normal distribution:

$$(11) \qquad \log(\tau_{itj}) \sim N(\bar{\tau}_{itj}, \sigma_{\tau}^2), \quad \log(\eta_{is}) \sim N(\bar{\eta}_{is}, \sigma_{\theta}^2).$$

To take into account the impact of assets on a household's starting probabilities in state s, we defineas a function of a household's total experience with financial products and the amount of financial assets at time 0. That is,

$$(12) \qquad \bar{\eta}_{is} = \omega_{0i} + \omega_{1i}TACCT_{i0} + \omega_{2i}ASSET_{i0},$$

where $TACCT_{i0}$ and $ASSET_{i0}$ denote the total amount of financial product categories and assets household i owns at time 0. Coefficients ω_{1i} and ω_{2i} measure how the number of accounts and total assets at time 0 affect the probability that a household starts in state s.

Household Heterogeneity and Estimation

We index the parameters of our multivariate probit model by household i to reflect the heterogeneity in response. To demonstrate variation in these parameters across households, we adopt a hierarchical Bayesian approach (Allenby and Rossi 1999; Heckman 1981). Specifically, let $\theta_{ijt} = [\beta_{0ij(s)}, \beta_{1i(s)}, ..., \beta_{10i(s)}]'$ be the vector of all the parameters in Equation 2 for household i, product j, and state s. We stack this vector across the products and states to yield a vector of all parameters for a given household: $\Theta_i = [\theta'_{i11}, ..., \theta'_{iJS}]'$. We use a linear model that relates demographic variables such as age and gender of the account holder and household size to values of these parameters. Formally, for θ_{im}, the mth element of Θ_i, for state s follows this model:

$$(13) \; \theta_{im} = \mu_{m0} + \mu_{m1}AGE_i + \mu_{m2}GENDER_i + \mu_{m3}HSIZE_i + e_{im}.$$

We assume $e_i \sim N[0, \Omega]$, where Ω is an $M \times M$ variance–covariance matrix.

To account for the possibility that the bank relies on endogenous information (demographics and product ownership) when generating cross-selling solicitations, we follow the approach Manchanda, Rossi, and Chintagunta (2004) propose. Specifically, we allow the observed cross-selling solicitaton to be a function of households' response parameters for several variables such as the number of accounts, age, income, and so on. To estimate our proposed model, we employ a Markov chain Monte Carlo (MCMC) approach, because the likelihood function involves high-dimensional integrals. The Web Appendix (http://www.marketingpower. com/jmraug11) provides a detailed explanation of the endogeneity issue, likelihood function, normalization, identification, and estimation of our model.

DYNAMIC OPTIMIZATION FRAMEWORK

Our multivariate probit customer response model incorporates dynamic components, which mean that a household's response to a cross-selling solicitation will vary depending on its current financial state and the cumulative effect of past solicitations. For a firm to maximize its profits, they must understand that solicitations may result in immediate purchases but also influence the future state of their customer, which in turn influences future responses. We propose a parsimonious method to obtain the answer: is to treat cross-selling decisions as solutions to a stochastic dynamic optimization problem.

Specifically, we let the indicator value Z_{ijkt} designate cross-selling solicitations, where Z_{ijkt} denotes the number of solicitations sent to household i for product j during period t using channel k (k = 1 for postal mail and k = 2 for e-mail):

$$(14) \quad Z_{ijkt} = \begin{cases} 1 & \text{if solicitation is sent to customer } \mathbf{i} \text{ for} \\ & \text{product } j \text{ through channel } k \text{ at time } t \\ 0 & \text{otherwise} \end{cases}.$$

In other words, the manager makes the promotion or solicitation decision about when (t) to send what product (j) to which customer (i) through which communication channel (k).

Expected Customer Long-Term Profit

The bank needs to evaluate the dynamic impact of current cross-selling solicitations on households' future profit contributions. Let $E[\Pi_{it} \mid Z_{ijkt}]$ be the expected profit earned across all financial products for household i during period t:

$$(15) \quad E[\Pi_{it} \mid Z_{ijkt}] =$$

$$\sum_{s=1}^{S} \left\{ \text{Prob}_{it}(s) \times \left(\sum_{j=1}^{J} [\text{Prob}(Y_{ijt} \mid s) E(BAL_{ijt}) r_{ij} - \sum_{k=1}^{K} c_k Z_{ijkt}] \right) \right\},$$

where $\text{Prob}_{it}(s)$ is the probability of household i being in state s during period t; $\text{Prob}(Y_{ijt} \mid s)$ is the predicted probability of household i purchasing product j at time t conditional on being in financial state s as defined by Equation 4; r_{ij} is the profit margin associated with each unit of balance of product j, which is assumed to be known; c_k is the unit cost of a cross-selling campaign through communication channel k; and $E(BAL_{ijt})$ is the expected balance household i for product j at time t the firm needs to predict when making decisions at time t. The Web Appendix (http://www.market-ingpower.com/jmraug11) explains the balance predictions.

Dynamic Cross-Selling Campaign Decisions

The bank's objective for its cross-selling campaign is to maximize the expected discounted profits from each household over the planning horizon.[4] Suppose the bank is interested in a planning horizon that begins in period ξ_1 and ends in period ξ_1 and the monthly discount rate is d, we can compute the expected discounted profits as follows:

$$(16) \quad \max_{\{Z_{ijkt} \mid t \in (\xi_1, \xi_2)\}} \sum_{t}^{\xi_2} (1 + \delta)^{t - \xi_1} E\{\Pi_{it} \mid [Z_{ijkt} \mid t \in (\xi_1, \xi_2)]\}.$$

The endogenous state variables are customers' financial states and the predicted purchase probability of products. All the endogenous financial states and exogenous state variables thus drive the optimal allocation decision, which is also the solution to the following Bellman equation:

$$(17) \quad V_{it} = \max_{\{Z_{ijkt} \mid t \in (\xi_1, \xi_2)\}} E\{\Pi_{it} \mid [Z_{ijkt} \mid t \in (\xi_1, \xi_2)]\}$$

$$+ \delta E[\max V_{it+1}(Z_{ijkt+1})] + \tau_{ijkt},$$

where $V_{it+1}(Z_{ijkt+1})$ is the expected optimal utility beginning from time t + 1, and τ_{ijkt} is the error term denoting unobserved factors affecting bank's solicitation decisions (Erdem, Imai, and Keane 2003; Erdem and Keane 1996; Sun 2005).

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[4]In the simulation, we assume the bank uses the observed data period of 27 months as the planning horizon for its cross-selling campaign. This is somewhat consistent with Gupta and Lehmann (2005) and Kumar et al. (2008), who use three years to calculate customer long-term value.

We define $\overline{V}_{it}(Z_{ijkt})$ as the deterministic part of the value function in Equation 17. To compute a solution, we assume τ_{ijkt} has an i.i.d. extreme value distribution, so we obtain logit choice probabilities for making solicitation decisions (Z_{ijkt}):

$$(18) \quad \Pr(Z_{ijkt}) = \frac{\exp[\overline{V}_{it}(Z_{ijkt})]}{\sum_{j, k} \exp[\overline{V}_{it}(Z_{ijkt})]}.$$

To overcome the challenge of large space, we adopt the interpolation method Keane and Wolpin (1994) propose and approximate values for the expected maxima at any other state points for which values are needed.

EMPIRICAL RESULTS

Model Comparison

We compared our estimated customer response model against five benchmark models to investigate the contribution of latent financial states, the long-term indirect roles (educational and advertising) of cross-selling campaigns, and heterogeneous channel preferences to predict customer purchase behavior. Model A is the latent financial maturity model Li, Sun, and Wilcox (2005) propose, which ignores the long-term roles of cross-selling and customer's channel preference, and assumes that latent financial maturity is linearly determined by household account ownership and experiences. Model B is the joint model of purchase timing and product category choice Kumar, Venkatesan, and Reinartz (2008) propose. In this model, customer category purchase choice is conditional on purchase timing while ignoring the long-term roles of cross-selling. These two benchmark models represent the most recent cross-selling models proposed in the marketing literature. Model C is our proposed model without latent financial states, long-term effects of solicitations, and heterogeneous preference for communication channels. Model D adds latent financial states to the third model. However, we do not allow long-term effects of solicitations or heterogeneous channel preferences. Model E adds long-term roles of advertising and education to Model D but not heterogeneous preferences for communication channels. Model F is our proposed customer response model, which nests Models C, D, and E as special cases.

To determine the number of states, we estimated models with between one and four states, the results of which are reported in Table 3. We find that the three-state version of the proposed Model F is the best-fitting model; therefore, we only report the three-state version for Model F. Table 4 reports the log of the marginal density (Chib and Greenberg 1995; Kass and Raferty 1995) and the hit rates of product purchases for the six models. The overall hit rate demonstrates how well our model can predict future customer responses. To forecast future observations, we calculate NACCT at time t + 1 as the sum of NACCT at t and the predicted new purchases at t, simulate the waiting time from Equation 6, and condition on other covariates. However, all models have access to the same information to preserve comparability across the forecasts.

Because consumer purchase occurs infrequently (approximately 3.1% of observations are purchases; see the sum of purchase transactions reported in Table 2), a naive predictor of no purchase would be correct 96.9% of the time. (Note that all our models do better than this naive prediction, with

Table 3

PROPOSED CUSTOMER RESPONSE MODEL WITH VARIOUS STATES[a]

		Hit Rate								
		Estimation Sample			Cross-Sectional Validation Sample			Longitudinal Validation Sample		
States	Log-Marginal Density of Estimation Sample	Overall	Purchase[b]	Nonpurchase[b]	Overall	Purchase	Nonpurchase	Overall	Purchase	Nonpurchase
One	−1,154,047.7	.980	.354	.981	.603	.193	.616	.800	.343	.806
Two	−1,144,416.0	.991	.355	.993	.606	.241	.618	.817	.346	.823
Three	−984,746.9	.995	.423	.996	.703	.268	.717	.832	.418	.837
Four	−1,184,113.2	.994	.319	.996	.613	.225	.625	.818	.301	.825

[a]For our proposed model and Models D and E, we must determine the number of latent states. To address this empirical question, we estimate the three competing models assuming one, two, three, and four states ($S = 1, 2, 3,$ and 4). The results for all four models show that three states result in the best-fitting model. For simplicity, we only present the results for the proposed model with various numbers of states in Table 3.

[b]We predict purchase without knowledge about the true predictive state and then separately report the accuracy of the predictions conditional on purchase and no purchase occurring. Note that we do not use the information that a purchase has occurred or not occurred when making the predictions.

Table 4

MODEL COMPARISON

Sample	Hit Rate	Model A: Li, Liechty, and Montgomery (2005)	Model B: Kumar, Venkatesan, and Reinartz (2008)	Model C	Model D	Model E	Model F: Proposed Model
Estimation	Log-marginal density	−1,192,557	−1,192,267	−1,210,556	−1,192,267	−1,192,179	−984,746
Hit rates	Overall	.985	.984	.973	.987	.987	.995
	Purchase	.141	.181	.151	.272	.294	.423
	No purchase	.997	.996	.985	.997	.997	.996
Cross-sectional validation	Overall	.623	.623	.583	.624	.635	.703
	Purchase	.125	.161	.101	.231	.246	.268
	No purchase	.639	.638	.598	.626	.647	.717
Cross-time validation	Overall	.757	.751	.698	.776	.810	.832
	Purchase	.140	.175	.150	.272	.295	.418
	No purchase	.765	.759	.705	.783	.817	.837

Notes: Model A = the latent financial maturity model; Model B = the joint model of purchase timing and product category choice; Model C = proposed model without financial state, indirect roles, channel preference, or endogeneity; Model D = proposed model with financial state but without indirect roles or channel, preference, or endogeneity; Model E = proposed model with financial state and indirect roles but without channel preference or edogeneity; and Model F = proposed model.

performance between 97.3% and 99.5%.) To create a more challenging predictive task, we report the accuracy of these predictions for purchase and nonpurchase observations separately.[5] The comparison of model fit and predictions across both the calibration sample and the two validation samples shows that our proposed Model F significantly outperforms the benchmark models, especially Models A and B. These results suggest that the innovations our customer response model provide are important.

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[5]We predict purchase without knowledge about whether purchase has occurred and then report the hit rates separately for the purchase and non-purchase observations. For our multivariate choice model, we must predict both when the purchase is going to occur and what is going to be purchased. This is different from multinomial choice model, which only concerns itself with the latter. Thus, our overall hit rate provides a measure of performance of incidence, while the hit rates for the purchase and nonpurchase samples measure accuracy of what is purchased. Consider the poorest performing Model A, which has a hit rate of 14.1% for the purchase sample, marginally worse than a naive model, which would predict purchase type correctly 14.3% of the time. (We got this percentage by the taking the average of the relative frequency of the type of product purchased from Table 2.) However, Model A still has a superior overall hit rate of 98.5%, substantially better than the naive prediction of always guessing no purchase; this would yield only a 96.9% accuracy (i.e., 100% less the observed purchase frequency of 3.1%). Therefore, a gain in accurately predicting when purchase occurs yields some trade-off in accuracy of detecting what is going to be purchased.

Parameter Estimates

Starting and transition probability equation. First, consider the parameters of the starting probability and transition probability functions in Table 5. We find that the probability that a household begins in a higher financial state (Equation 12) increases with more accounts or more assets deposited with the bank during the initial period, consistent with our intuition. Similarly, the estimated hyperparameters for such states with the transition probabilities (i.e., $\bar{\tau}$ in Equation 11) indicate that when a household switches states, it is more likely to switch to a higher state (i.e., State 2 or 3) than a lower one (see the larger hyperparameter estimates in higher states, p-value = .001 or 0 for States 2 and 3, respectively).[6]

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[6]The p-values reported in this section refer to the probability that a one-sided test will have differences between the coefficients different than zero. They are computed using the empirical probability of the difference being negative from our MCMC samples, which appropriately marginalizes across the uncertainty of the parameters. The p-values are small because the data are able to differentiate between the financial states and large number of observations provides strong information in making the inferences. However, the sampling error in our MCMC estimates indicates the p-values have some chance of being higher than the 0 or .001 reported but are clearly highly significant (<.01).

Table 5

ESTIMATION RESULTS FOR THE HOUSEHOLD CHOICE MODEL

Explanatory Variables	State 1	State 2	State 3
Starting Probability Model			
Intercept	–28.38* (4.27)	–11.55* (4.35)	10.25* (3.15)
TACCT	–16.17* (6.34)	39.19* (5.03)	6.55 (4.25)
ASSET	–39.16* (2.80)	–31.83* (2.46)	4.69 (3.77)
Transition Probability Model			
Hyperparameter $\bar{\tau}$	–1.39* (.11)	–.69* (.13)	.74* (.22)
Gamma Scale Parameters $k_i(s)$			
$k_i(s)$	6.37* (1.17)	5.04* (1.22)	1.12* (.14)
Waiting-Time Model			
Intercept	–8.23* (4.82)	–8.53* (4.89)	13.43* (1.70)
TACCT	–16.38* (2.99)	12.51* (2.81)	.27 (2.12)
Number of solicitations of:			
Checking	–14.12* (4.53)	14.87* (2.75)	19.12* (4.48)
Saving	–7.52* (3.91)	37.66* (2.43)	–1.20 (2.93)
Credit/bank			
Cards	–24.72* (3.73)	31.46* (7.97)	–.96 (6.00)
Loans	11.13* (4.86)	–15.18* (3.00)	–22.89* (3.55)
CDs	7.76* (2.83)	–8.28* (.73)	–4.08 (3.83)
Investment	19.38* (3.05)	27.45* (2.76)	–39.30* (3.09)
Others	–29.05* (1.54)	–14.40* (2.90)	30.66* (3.39)
Cumulative mail solicitations	6.02 (6.19)	–8.18* (2.82)	–1.81 (1.92)
Cumulative e-mail solicitations	–30.28* (4.25)	–20.02* (2.40)	–12.71* (1.98)
Square of cumulative mail solicitations	10.58* (3.43)	–.20 (2.03)	10.01* (3.68)
Square of cumulative e-mail solicitations	4.91 (3.84)	22.57* (1.84)	6.72* (2.58)
COMP	9.27* (3.52)	26.90* (4.59)	3.40 (5.82)
Income	–14.02* (4.09)	–3.64 (5.69)	18.11* (2.08)
CLOSE	13.12* (2.98)	–11.32* (3.54)	–17.31* (5.06)
Utility Purchase Model			
Product-specific intercepts			
Checking	–3.46* (.07)	.25* (.04)	4.04* (.09)
Savings	–3.27* (.12)	–.13* (.06)	3.06* (.05)
Credit/bank cards	–2.88* (.09)	–.78* (.05)	3.99* (.04)
Loans	–3.47* (.09)	.15* (.04)	3.12* (.05)
CDs	–3.63* (.24)	.26* (.02)	2.98* (.21)
Investment	–4.30* (.03)	.04 (.09)	5.48* (.09)
Others	–3.48* (.05)	.06 (.06)	3.42* (.07)
Number of solicitations of:			
Checking	.65* (.01)	.69* (.01)	.73* (.01)
Savings	.41* (.02)	.44* (.02)	.54* (.03)
Credit/bank cards	.52* (.02)	–.71* (.04)	–.74* (.04)
Loans	.12* (.01)	.70* (.02)	.05* (.01)
CDs	.43* (.01)	.51* (.01)	.50* (.01)
Investment	.36* (.01)	.43* (.02)	1.52* (.01)
Others	.90* (.03)	1.19* (.06)	.12* (.03)
Number of solicitations of:			
Mail	.19* (.02)	.21* (.01)	.29* (.02)
E-mail	.34* (.03)	.52* (.02)	.50* (.02)
Lag of cumulative total solicitations	.23* (.01)	.34* (.01)	.43* (.01)
$\mu_{\Delta BAL_{it-1}}$	1.06* (.02)	1.29* (.05)	1.24* (.04)
$\sigma^2_{\Delta BAL_{i(t-1)}}$	–1.45* (.06)	–1.92* (.09)	–1.86* (.08)
NACCT	–.22* (.01)	.24* (.01)	.29* (.01)
TRANS	.24* (.01)	.33* (.02)	.32* (.03)
TENURE	.04* (.02)	.04* (.01)	–.01 (.01)
COMP	–.05* (.01)	–.09* (.02)	–.16* (.01)
Income	.34* (.01)	.40* (.02)	.45* (.02)

*Significant at the 95% probability level.

Waiting time equation. In the expected-waiting-time Equation 7, the constant terms in the waiting-time function are estimated to be –8.23, –8.53, and 13.43 for the three states. The ordering of the coefficients (negative constants in States 1 and 2) indicates that households have an intrinsic preference to stay in State 3 for a longer time (*p*-values are .001). The coefficient of the number of accounts in State 1 is negative and significant, implying that households with

more financial products are less likely to stay in State 1, while those with more are more likely to stay in State 2 or 3.

Comparing the product-specific coefficients on the number of solicitations, we find that solicitations that promote checking, savings, other, and credit cards in the first state, those that promote loans and CDs in the second state, and those that promote investment and loans in the third state encourage customers to stay for a shorter period and to move faster along the financial state continuum (e.g., *p*-

value = 0 for comparing investment coefficient with checking coefficient in the third state). This supports our contention that offering the right product is important, because checking account solicitations are helpful in decreasing the customer's time in the first state. This also illustrates that states are not solely determined by exogenous financial conditions (e.g., customer's age, income), but also marketing activity by the bank.

Comparing the coefficients of e-mail and postal mail solicitations, we find that the educational role is higher (more negative; p-values = .001 for all three states) when the bank uses e-mail than when it uses postal mail, possibly due to the rich information and interactive nature of e-mails (Ansari and Mela 2003). However, the positive coefficients of the squared terms of these two variables indicate that receiving too many solicitations reduce the effectiveness of the educational role of campaigns, which agrees with findings in Venkatesan and Kumar (2004) and Venkatesan, Kumar, and Bohling (2007). This result is consistent with our conjecture that too many solicitations wear out a customer's attention, thereby reducing the marginal educational role.

Therefore, our results confirm the educational role of solicitations in helping households move faster along the financial continuum when a bank solicits households on checking, savings, others, and credit cards in the first state, loans and CDs in the second state, and investments and loans in the third state. The educational role differs across communication channels and products. It is more effective when a bank uses e-mail than when it uses postal mail. However, the educational role wears out when a bank sends too many solicitations to the same household. It is significant that we also find that the more accounts households close, the longer they stay in the first state and the shorter they stay in the higher states (p-value = .001 and 0 for the second and third states, respectively).

Purchase equation. Table 5 indicates the estimates of the coefficients in the purchase utility model, and Table 6 indicates the error correlation matrix. According to the magnitude (from high to low) of the estimated product-specific intercepts, we find that households in the first financial state have an intrinsic preference for credit cards, checking, and savings, followed by loans, others, CDs, and investments. In the second state, the ranking is CDs and loan products, checking, investment, others, savings, and credit cards. In the last state, the ranking is investment, checking, credit cards, others, loans, savings, and CDs (e.g., p-value = 0 for comparing investment coefficient with checking coefficient in the third state).

The coefficients of the solicitations in the current month measure the instantaneous effect of promotions.[7] Comparing the product-specific solicitation effect, we show that the instantaneous promotional effects are higher for checking, savings, credit cards, and others in the first state; for loans, CDs, checking, and others in the second state; and for investment, checking, and CDs in the third state (e.g., p-value = 0 for comparing checking coefficient with saving coefficient in the first state). The cumulative solicitations up to the current month also significantly increases the likelihood the household will open new accounts. Households are likely to view receiving more solicitations as a signal of customer care and relationship building and thus are encouraged to open new accounts with this bank.

Both postal mail and e-mail solicitations in the current month as well as solicitations for each financial product increase the likelihood the household will open a new account for all three states. Note that both postal mail and e-mail solicitations are slightly more effective in higher states (i.e., the second and third states; p-value = .001 or 0 when comparing the third state with the first state for mail and e-mail, respectively) because households in higher states may be more financially mature and may have stronger relationships with the bank, thereby engendering trust and making them more responsive to cross-selling solicitations (Kamakura, Ramaswami, and Srivastava 1991).

As expected, the positive coefficient on the mean change of financial assets increases the probability of a household opening a new account with the bank. However, the variance of change of total assets in the bank decreases the purchase probability. This result may have occurred because the higher the mean of the balance change, the more assets are available, and a higher variance means less financial stability (Li, Sun, and Wilcox 2005). It is noteworthy that owning more accounts in other product categories decreases the purchase probability of the focal category in the first state but increases the purchase propensity in the second and third states. This may be because customers in the low financial state may have more financial resource constraints

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[7]In our model, the coefficients of solicitations in the purchase utility model measure the responsiveness conditional on the household is in a particular state. The reason that our model results in more significant coefficients is that by taking into account intracustomer heterogeneity or evolvement of financial states, we recognize the situations when households are not ready for a particular financial product and thus are not responsive to the cross-selling solicitations. However, this cannot be captured by models ignoring the evolvement of financial states. The same coefficient is estimated to be insignificant. Indeed, most parameters in Model C (the benchmark model ignoring indirect effects of solicitations) are not significant.

Table 6

ESTIMATION RESULTS FOR THE CORRELATION MATRIX OF THE HOUSEHOLD ACCOUNT CHOICES

Product	Checking	Saving	Credit Cards	Loans	CDs	Investment	Others
Checking	1						
Savings	.06* (.01)	1					
Credit cards	.11* (.01)	.07* (.01)	1				
Loans	.03 (.05)	.02 (.06)	−.03 (.03)	1			
CDs	.01 (.01)	.03 (.06)	.06 (.05)	−.07 (.10)	1		
Investment	.06* (.03)	−.05 (.05)	.10* (.01)	−.04 (.05)	.05* (.01)	1	
Others	.09* (.03)	.07* (.01)	.23* (.05)	.04 (.04)	.08* (.01)	.13* (.01)	1

*Significant at the 95% probability level.

or lower commitment to the bank than those in higher states, which results in higher intercategory competition.

Customers with a higher number of cumulative transactions are more likely to open new accounts with the bank because of the strengthening of the customer–seller relationship (Anderson and Weitz 1992; Kalwani and Narayandas 1995). Tenure—measured by the length of time a household has been a bank customer—increases the switching cost and thus the likelihood of opening new accounts. A higher percentage of assets in other financial institutions decreases the probability of the customer purchasing new accounts. Higher income increases the purchase propensity in all three states (Li, Sun, and Wilcox 2005; Paas, Bijmolt, and Vermunt 2007).

This ranking of the instantaneous solicitation effectiveness in the utility function is roughly consistent with that of the educational effectiveness in the expected-waiting-time equation. It is also similar to the ranking of the constant terms in the utility equation that indicate household intrinsic preference. The results imply that households have different priorities for various financial products during each financial state. In the first state, they demonstrate a higher preference for checking, savings, and credit cards or products that provide financial convenience and are more likely to respond to solicitations of these products. In the second state, they prefer and are more likely to respond to solicitations selling loans and CDs, which reflect their need for financial flexibility. In the third state, they prefer and are more likely to respond to solicitations selling investment-related products. On the basis of the products customers are more likely to buy and their responsiveness to the cross-selling campaigns in each state, we term the three states as a convenience state, a flexibility state, and a growth state.

Household heterogeneity. Table 7 reports the estimation results for the hierarchical component of the utility equation. Most of the significantly estimated coefficients have the expected signs and demonstrate that the instantaneous solicitation effect varies across households according to their characteristics. Consider gender as an example. Note that balance increases purchase probability more for male

customers, while tenure effects show that men are not as likely to remain loyal, perhaps because male customers are more likely to take advantage of competitive offers from other financial institutions (Barber and Odean 2001). Male customers are also less likely to respond to investment and loans solicitations, perhaps because they believe themselves to be more knowledgeable about the financial products and hence more confident in managing their investments (Barber and Odean 2001).

Hidden Markov process. Tables 8 and 9 present the estimation results for the HMM. Table 8 shows that a household is most likely to start in a convenience state (first state) or in a growth state (third state), with 32% and 57% probability, respectively. We compute the average waiting times for each state to be 9.68, 10.90, and 15.07 for s =1, 2, and 3, respectively, based on Equation 7. Table 9 lists the transition probabilities for the HMM. Note that households in our study tend to have a higher probability of switching to the convenience state (first state). For example, if a household is currently in the second state, the transition probability from the second to the first state is 93%, and it is 7% for switching to the third state. Moreover, if a household is currently in the third state, we estimate it has a 92% chance of switching from the third state to the first state and an 8% probability of switching from the third state to the second state.[8] Consistent with our finding in the waiting-time model, this may indicate the first state represents a quiet attrition state in which households have low financial maturity and gradually close accounts.

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[8]Our proposed customer response model is general enough to allow the possibility for customers to move freely back and forth among states. A nested version of our proposed customer response model can constrain customers to move up only from State 1 to State 3. In our applications, the results show the general trend of consumers to sequentially migrate up from State 1 to State 2 to State 3, with some households estimated to go back and forth (approximately 5.81% of the households). Reversion from more advanced states to earlier states may be the result of consumer attrition, changes in their financial status, repeat purchases for other household members, or the aggregation of product variations.

Table 7

ESTIMATION RESULTS OF HOUSEHOLD HETEROGENEITY

Covariates	Intercept		Age		Gender (Male)		Household Size	
Number of Solicitations of:								
Checking	.60*	(.09)	−.01	(.01)	.11*	(.05)	.06*	(.03)
Saving	.31*	(.11)	.01	(.01)	−.08	(.05)	.01	(.02)
Credit/bank cards	.51*	(.10)	.01	(.01)	.01	(.06)	−.01	(.02)
Loans	.25*	(.12)	−.01	(.01)	−.17*	(.06)	−.01	(.02)
CDs	.30*	(.13)	.01	(.01)	.12*	(.05)	−.02	(.02)
Investment	.47*	(.12)	−.01	(.01)	−.05*	(.01)	−.01	(.03)
Others	.81*	(.12)	.01	(.01)	−.02*	(.01)	−.04*	(.02)
Number of Solicitations of:								
Mail	.25	(.14)	.01	(.01)	-.06	(.05)	−.01	(.02)
E-mail	.44*	(.11)	−.03*	(.01)	.11*	(.05)	−.04*	(.02)
Lag of cumulative total solicitations	−.03	(.10)	−.01	(.01)	.06	(.05)	−.05*	(.02)
$\mu_{\Delta BAL}$	.83*	(.08)	.01	(.01)	.06*	(.02)	.02*	(.02)
$\sigma_{\Delta BAL}$	−1.39*	(.10)	−.01	(.01)	−.03	(.01)	−.01	(.02)
NACCT	.21*	(.10)	.01	(.01)	−.04	(.05)	.01	(.02)
TRANS	−.22*	(.10)	−.01	(.01)	-.07	(.05)	.04*	(.02)
TENURE	.01	(.09)	.01	(.01)	−.10*	(.05)	−.01	(.02)
COMP	−.08	(.11)	.02*	(.01)	−.03*	(.01)	.05*	(.02)
Income	.27*	(.13)	.01	(.01)	-.04	(.05)	.04*	(.02)

*Significant at the 95% probability level.

Table 8
ESTIMATION RESULTS FOR THE HMM

State	Starting Probability	Waiting Time
State 1	.32* (.03)	9.68* (.04)
State 2	.11* (.01)	1.90* (.05)
State 3	.57* (.03)	15.07* (.24)

*Significant at the 95% probability level.

Financial States

In this section, we investigate whether and how customers move along a financial continuum over time. In Figure 1, we plot the average probabilities of customers residing in the three stages against time. We compute these probabilities using a filtering approach to recover the person's state at any given time period (Montgomery et al. 2004; Netzer, Lattin, and Srinivasan 2008). We find customers tend to slowly move through time from the first state to the second state, and then to the third state. In other words, customers begin in a financial state in which they are more likely to look for convenience, move to a state in which they need financial flexibility, and then to a state in which they seek riskier growth investments.

Decomposition of Long-Term Solicitation Effects

Given that cross-selling solicitations have demonstrated their instantaneous, advertising, and educational roles, it is interesting to measure their relative strength. We arbitrarily chose a month (Month 3) during which little cross-selling solicitation occurred and chose loans as a cross-selling solicitation example. We increased by 10% the frequency of households receiving loan solicitations through postal mail and randomly selected the recipients. Using the posterior estimates of the proposed customer response model, we report the probability changes of being in each of the three financial states in Columns 2–4 of Table 10. For example, an increase in loan solicitations during Month 3 results in a .90% increase of being in State 2 but a decrease of –.44% and –.46% of States 1 and 3, respectively. We also find that there is an instantaneous increase in purchase probability of loans of .30%, which is listed in the column titled "Change

Figure 1
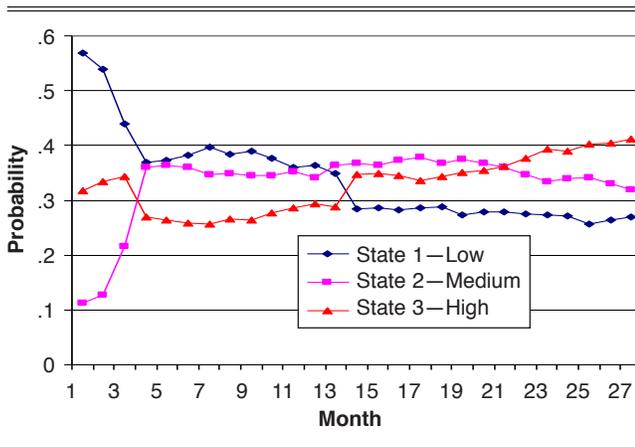### EVOLVING OF CUSTOMER FINANCIAL STATES OVER TIME

Table 9
ESTIMATION RESULTS FOR THE TRANSITION PROBABILITY MATRIX OF THE HMM

State	State 1	State 2	State 3
State 1	0	.20* (.01)	.80* (.01)
State 2	.93* (.01)	0	.07* (.01)
State 3	.92* (.02)	.08* (.02)	0

*Significant at the 95% probability level.

of Probability of Purchasing" in Table 10 (the numbers in the table are percentages).

The educational role of cross-selling occurs through the HMM process, specifically by influencing the consumer's switching to different financial states in the future. If we ignore the probability of state changes and compute the effect of our increasing loan cross-selling, we can estimate the direct effect of cross-selling promotions separately from the educational effect on the purchase probability of loans. Our estimate of this direct effect of cross-selling on loan purchase probability is given in Table 10 (the column titled "Direct Effect"). Initially, in Month 4, the increase in purchase probability of loans is .14%, but by Month 27 it drops to .02%. Overall, this increases a household's cumulative purchase probability of loans by 1.88% from Month 4 to Month 27.

If we consider the state changes (e.g., which includes the educational role of cross-selling through its influence on the state changes of the HMM), we find there is a much greater impact on loan purchases from our hypothetical loan solicitation. Starting from the third month, we note the probabilities of households belonging to the second state (financial flexibility state) increase, whereas those of the first state decrease (those of the third state first increase and then decrease). This means the increase in loan solicitations in Month 3 speeds up household movement along the financial maturity continuum toward the flexibility state (State 2). Thus, over the course of Months 3–27, we find a cumulative 12.72% increase in a consumer purchasing a loan. Among this increase, only 2% (.003/.127) is due to the instantaneous promotional effect, 15% (= .019/.127) is due to the lasting advertising effect, and 83% [(.127 – .003 – .019)/.127] comes from an educational effect. Thus, in this example, the educational role of cross-selling solicitations largely dominates the direct effects, which include the instantaneous promotional and advertising effects.

SIMULATING CUSTOMER-CENTRIC CROSS-SELLING SOLICITATIONS

Dynamic and Customized Solicitations

On the basis of the estimated parameters, the observed history, and customer demographic variables, we simulate optimal solicitation decisions (Z_{ijkt}^*) using our proposed dynamic programming framework (Equations 14–18). We obtain a sequence of cross-selling campaign decisions Z_{ijkt} about when (t) to send out solicitations to which households (i) to cross-sell which product (j) using which communication channel (k). To succinctly demonstrate how the solicitations decisions are driven by financial states, in Figure 2, Panel A, we draw the average probabilities of sending cross-selling campaigns on the J products $[1/(I \times K \times T)]\Sigma_{i=1}^{I}\Sigma_{k=1}^{K}\Sigma_{t=1}^{T}$

Table 10

DECOMPOSITION OF THE EFFECTS OF CROSS-SELLING SOLICITATIONS IN PERCENTAGES

| | Proposed Model | | | | |
| | Change of Probability | | | | Change of Probability of Purchasing |
Month	State 1	State 2	State 3	Direct Effect	
3	−.44	.90	−.46		.30
4	−.81	.58	.23	.14	.66
5	−.77	.49	.28	.14	.48
6	−.92	.50	.42	.12	.45
7	−.95	.53	.42	.17	.57
8	−1.01	.52	.49	.15	.33
9	−.79	.38	.41	.13	.59
10	−.74	.37	.37	.10	.52
11	−.81	.48	.33	.09	.69
12	−1.00	.46	.54	.09	.56
13	−.99	.56	.43	.09	.71
14	−.72	.43	.29	.08	.58
15	−.61	.60	.01	.07	.65
16	−.45	.41	.04	.06	.60
17	−.51	.33	.18	.06	.33
18	−.33	.50	−.17	.06	.33
19	−.24	.41	−.16	.05	.54
20	−.14	.42	−.28	.05	.54
21	−.14	.63	−.49	.04	.46
22	−.15	.68	−.53	.04	.25
23	−.27	.71	−.44	.04	.77
24	−.14	.61	−.47	.03	.39
25	.01	.69	−.70	.03	.35
26	−.09	.62	−.54	.02	.71
27	.34	.50	−.84	.02	.35
Cumulative	−12.66%	13.32%	−.66%	1.88%	12.72%

$Pr(Z^*_{ijkt})$ against the three states. As shown in Figure 2, Panel A, our proposed cross-selling campaigns are developed according a customer's financial maturity state. For example, the probability of sending out convenience-related financial products (e.g., checking and saving accounts) is highest in the first state, and the probability of sending out more complex products, such as loans and CDs in the second state and investments in the third state, is the highest.

According to the findings from Figure 1 and Figure 2, Panel A, during earlier observation periods that correspond to the earlier financial stages of an average customer in our sample, we recommend solicitations for checking and savings. During the middle observation period, our proposed solution suggests sending this customer promotions that involve CDs and lending-related financial products. During the latter part of our observation period, the solution recommends sending out investment-related products. Thus, our proposed solution is dynamic in that the decisions of when and which products to send solicitations are made in accordance with the household's evolving financial maturity state.

Next, we use age as an example to show how the proposed solicitations are customized according to customer heterogeneity and channel preference (whom to send the solicitations and which channel to use). In Figure 2, Panel B, we again take loans as an example and plot the probability of sending a solicitation, given by $(1/T)\Sigma^T_{t=1}Pr(Z^*_{ijkt})$, for this product against age. To demonstrate whether the cus-
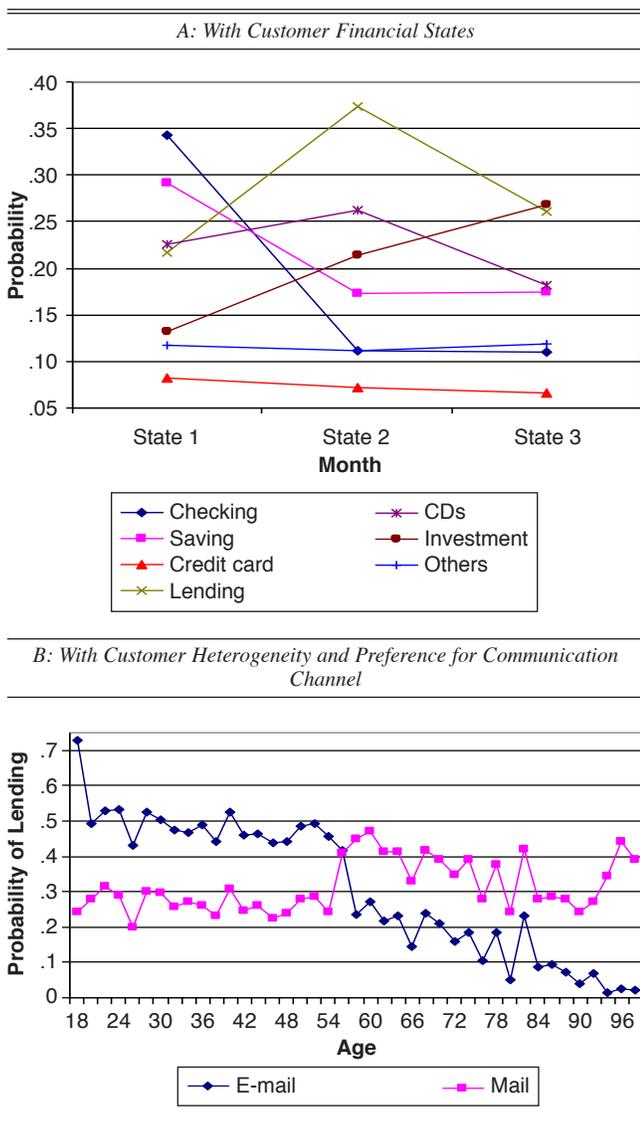
tomization differs across communication channels, we draw the curve for both e-mail and postal mail channels. This snapshot analysis allows us to show how the proposed solution is tailored to age. We show that the probabilities of sending out loan solicitations using postal mail to old customers (aged 50 years or older) are higher than for other customers. Note that the solution suggests the solicitation channel should be customized for demographics and channel preference: They should be sent through e-mail for younger customers and postal mail for older customers.

Improvement of Long-Term Response Rates and Profit over Alternative Frameworks

Finally, we compare the response rates of our proposed solicitation solutions with a few alternative approaches against those observed in our sample. In the first alternative framework, we follow conventional industry practices as observed in our data set and compute the sample product ownership transition matrix (e.g., the purchase probabilities conditional on owning a particular product). This sample transition matrix approach simply makes use of the observed purchase ordering (i.e., first-order product transition matrix) from the estimation sample to predict customers' purchases. For brevity, we refer to this as the campaign-centric approach.

In the second alternative framework, we estimate a logit model that is similar to existing cross-selling customer response models such as Li, Sun, and Wilcox (2005). This

Figure 2
CROSS-SELLING SOLICITATIONS CHANGE

A: With Customer Financial States

B: With Customer Heterogeneity and Preference for Communication Channel

approach assumes the latent financial maturity is linearly determined by household account ownership and experiences. We used logit models to predict the response rate. Those customers with the highest expected profit are offered the campaign. Thus, the solicitation decisions are made in a myopic way.

The third alternative framework is similar to Kumar, Venkatesan, and Reinartz (2008) in that it targets customers with the higher long-term value. We calculated customer long-term value as the net present value of the predicted stream of future profits. This framework does not account for intracustomer heterogeneity, nor does the bank employ dynamic programming to optimize future actions.

The fourth alternative approach follows a customer response model that ignores financial maturity, intracustomer heterogeneity and long-term effects of solicitations (Model C). The optimization framework is myopic and ignores customer life time value. The fifth and sixth alternative approaches allow the customer response model to take into

account both intracustomer heterogeneity and long-term effects of solicitations (Model F). The fifth framework assumes the bank is myopic, while the sixth framework incorporates CLV and follows our proposed dynamic optimization framework.

In Table 11, we report and compare the number of mail and e-mail solicitations sent out, the short- and long-term response rates, total profit, and return on investment (ROI) during our observation period using the calibration sample.[9] Note that our proposed framework does not result in the highest short-term response rate. Rather, our objective is to maximize the long-term response rate, which we find to be significantly higher than all the other techniques. Our gains occur by recognizing the financial readiness of a customer and long-term effects of solicitation on customer responses. The result is a sequence of solicitation decisions that maximize long-term customer response rate and profit. This means some solicitations are not sent to seek an immediate response but to help educate customers and prepare them for future solicitations. In addition, note that the total number of mail solicitations resulting from our dynamic optimization framework is about half of current practices as observed in the data. Thus, recognizing the customer's financial development reduces irrelevant messages.

Comparing the 5.1% response rate from cross-selling solicitations of the campaign-centric approach, the long-term response rate based on the proposed framework (Alternative 6) is 12.7%—a significant 149% (131%) improvement. Moreover, ROI improves by 78.1%, and the total profit improves by 177%. Similar comparison holds for the first alternative framework.

Both the immediate response rates and long-term response rates resulting from Li, Sun, and Wilcox (2005) and Kumar, Venkatesan, and Reinartz (2008) are improved over those observed in the sample and the first alternative. These two approaches improve over the first alternative approach because CLV is treated as another segmentation variable to differentiate profitable customers from unprofitable ones. However, the improvement of long-term response rate, total profit, and ROI are not as impressive as our proposed approach. This is because both frameworks ignore intracustomer heterogeneity and long-term effects of solicitations, and treating CLV as another segmentation variable is different from our proposed dynamic programming approach.

On the basis of individual customer response model, the fourth alternative improves over the campaign-centric approach because it allows for individual targeting. As expected, the fifth alternative framework results in higher short- and long-term response rates than those observed in the sample. This is because it allows the bank to follow the evolution of each household and makes a customized and dynamic solicitation schedule for each household. However, being myopic, this framework cannot be proactive in taking advantage of the long-term educational role. Thus, it results in lower long-term response rate compared with the proposed framework.

Our proposed framework (the sixth alternative) takes into account the development of customers, the educational role

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[9]We obtain similar results using the cross-sectional holdout sample. The improvement of ROI is 53.4%.

Table 11

COMPARISON OF ALTERNATIVE OPTIMIZATION FRAMEWORKS FOR CROSS-SELLING

Alternative Frameworks	Number of Mail Solicitations Sent Out	Number of E-mail Solicitations Sent Out	Short-Term Response Rate	Long-Term Response Rate	Total Profit (95% Confidence Interval)[a]	ROI
Campaign-centric (observed in the sample)	10,891	1699	.050	.051	22,021.5	35.33%
Alternative 1 (sample transition matrix approach)	9136	6700	.054	.055	23,179.5 (22,890.5, –23,468.5)	36.97%
Alternative 2 (Li, Sun, and Wilcox 2005)	9067	8078	.080	.081	32,739.0 (32,441.8, –33,036.2)	40.12%
Alternative 3 (Kumar, Venkatesan, and Reinartz 2008)	8503	8871	.086	.094	51,062.1 (50,695.5, –51,428.7)	41.64%
Alternative 4 (based on Model C and ignore state, long-term effects, and lifetime profit)	9093	7423	.065	.066	25,645.3 (25,088.7, –26,201.9)	38.21%
Alternative 5 (based on Model F and ignore lifetime profit)	7439	8500	.087	.103	51,374.2 (50,818.6, –51,929.8)	43.28%
Alternative 6: Proposed (based on Model F)	6730	6552	.078	.127	60,925.5 (60,228.5, –61,622.5)	62.92%

[a]The confidence intervals are calculated as the 95% probability intervals of the mean of the total profits based on the draws of our MCMC algorithm and refer to the confidence interval of the mean of total profits and not the confidence of total profits.

of solicitations in impacting future response, and the goal of maximizing long-term profit. The improvement of performance dominates all the other alternative decision frameworks. Comparing the magnitudes of improvements of Alternatives 4 and 6, we find that improvement in long-term response rate and total profit are highest when dynamic decisions are made, followed by proactive decision making and customization, respectively. The improvement on ROI is highest when decisions are made in a proactive decision making, followed by dynamic decisions and customization, respectively.

CONCLUSIONS, LIMITATIONS, AND FUTURE RESEARCH DIRECTIONS

Low response rates are challenging managers to improve the effectiveness of cross-selling campaigns. We believe current cross-selling focuses too much on individual campaigns and not enough on the dynamic effects inherent in a customer-centric approach. We find that cross-selling campaigns can be improved by understanding how cross-selling solicitations change customer purchase behavior and tailoring these campaigns to each customer's evolving needs and preferences to enhance long-term customer relationships and optimize long-term profits.

Using cross-selling campaigns and purchase history data provided by a national bank, we propose and estimate a customer-response model that recognizes latent financial maturity and preference for communication channels. Our results demonstrate that customer responses to cross-selling solicitations of different products do indeed evolve over time. In addition, cross-selling solicitations movie customers to a latent state when the customer prefers the cross-sold product (educational role) or building up a long-term relationship (advertising role). A decomposition study reveals that the educational effect dominates the instantaneous promotional and advertising effects. Furthermore, we find that relative to postal mail solicitations, e-mail solicitations are more effective at more advanced stages of customer financial maturity and are more effective at educating customers.

Using the estimated customer-response parameters, we formulate the bank's cross-selling decisions as solutions to a stochastic dynamic programming problem that maximizes customer long-term profit contribution. This proposed dynamic optimization framework results in a sequence of solicitations that represent an integrated multistep, multi-segment, and multichannel cross-selling campaign process to optimize the choice and timing of these messages. Compared with current practice observed in our data set, our proposed approach improves immediate response rate by 56%, long-term response rate by 149%, and long-term profit by 177%.

This is the first study to explicitly investigate how cross-selling solicitations dynamically interact with customer purchase decisions in the short and long run. It also establishes the importance for banks to take a long-term view and develop a proactive sequence of campaign massages to influence the growth path of households' financial maturity. Our dynamic programming approach serves as an analytical decision-making tool for analyzing rich customer databases and deriving concrete direct marketing solutions on how to target the right customer with the right product at the right time with the right channel. It also provides a computational algorithm for firms that are looking for one-on-one, interactive, and real-time marketing solutions enabled by current technology. Potentially simplified heuristics could approximate our decision rule. For example, the current practice of cross-selling financial products to customers according to a snapshot of their current demographics and product ownership approximates the customization property. However, this simplified heuristic does not well approximate the dynamic and proactive elements of our strategy and leaves

room for improvement by incorporating dynamic and proactive properties.

This research is subject to limitations and opens avenues for further research. First, our study is limited by a two-year history and lack of competition information. A sample with longer longitudinal data and more complete information on competitors' offers would expose the model to changing competitive conditions, economic cycles, and interest rates and more longitudinal variation in customer history. Second, many banks emphasize account acquisition and overlook retention of account balances. Further research could explicitly model account closings and account openings. A third direction for further research is to study the migration of service channels (Ansari, Mela, and Neslin 2008). Fourth, researchers could show how solicitations increase customer financial knowledge and explicitly test the educational role of solicitation. Finally, further research could take into account the effect of solicitations on usage, account balance, and customer retention, which is beyond the scope of our research

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