Duration

- Measure sensitivity of bond price to changes in interest rate
- Often used as a risk measure for a bond
- Another definition: Duration is the average time it takes the bond to return its price

Duration is associated with famous names in economics/finance

- Frederick Macaulay (1882–1970)

Basic duration computation

\[
\text{Duration} = \frac{1}{P} \sum_{t=1}^{N} \frac{tC_t}{(1+r)^t}
\]

where

\[
P = \text{bond price} = \sum_{t=1}^{N} \frac{C_t}{(1+r)^t}
\]

\[
C_t = \text{bond payment at time } t
\]

\[
r = \text{bond yield to maturity}
\]

Example: Bond A

- 10-year bond with 7% coupon
- Face value $1,000
- Issued at par
  - Meaning: price of bond = $1,000
  - Meaning yield to maturity (IRR) of bond is 7%
- Years 1–9: payment of coupon only (7% * 1,000)
- Year 10: payment of coupon + face value = $70 + $1,000
Bond A’s duration = 7.5152

Example: Bond B
- 10-year bond with 13% coupon
- Face value $1,000
- Current bond price = $1,421.41

\[
\frac{1}{P}\sum_{t=1}^{10} \frac{t \times C_t}{(1+r)^t}
\]

Meaning yield to maturity (IRR) of bond is 7%
- Years 1-9: payment of coupon only (13%*1,000)
- Year 10: payment of coupon + face value = $130 + $1,000

Bond B’s duration = 6.7535

Functions in Excel and in FM4
- Excel has a function called Duration
- Financial Modeling defines an alternative function called DDuration

Excel’s Duration function

**Duration** (settlement, maturity, coupon, yld, frequency, [basis])

**In Formula**: Dates should be entered by using the DATE function, or as results of other formulas or functions. For example, use DATE(2008,5,23) for the 23rd of May, 2008. Problems can occur if dates are entered as text.

The **DURATION** function syntax has the following arguments:
- **Settlement**: Required. The security’s settlement date. The security settlement date is the date after the issue date when the security is traded to the buyer.
- **Maturity**: Required. The security’s maturity date. The maturity date is the date when the security expires.
- **Coupon**: Required. The security’s annual coupon rate.
- **Yld**: Required. The security’s annual yield.
- **Frequency**: Required. The number of coupon payments per year. For annual payments, frequency = 1; for semiannual, frequency = 2; for quarterly, frequency = 4.
- **Basis**: Optional. The type of day count basis to use.
Duration applied to Bond A: Screens

Bond A has 10-year maturity. I've chosen the Settlement (starting) date and the Maturity dates to be 10 years apart. The Frequency refers to the number of payments per year (in our example: 1 payment/year).

What does duration mean?

- Duration can be used to approximate the change in bond price when the discount factor \(1+r\) changes

\[
\frac{\Delta P}{P} \approx -D \frac{\Delta r}{1+r}
\]

Using duration to approximate bond price changes

Duration as a function of coupon

Larger coupons mean that the bond returns its price earlier. Since Duration measures the average time that the bondholder has to wait to get his/her money back, larger coupons mean lower duration.