American vs European options

- European option gives right to buy/sell stock only on the exercise date $T$
  - If you don’t want to exercise this right, you don’t have to.
- American option gives the right to buy/sell stock on or before exercise date $T$

Notation

- $C$ for call
  - Time subscript if necessary: $C_0$ or $C_t$
- $P, P_0, P_t$ for put
- $X$ or $K$ for exercise price
  - Also called strike price
- $S, S_0, S_t$ for stock price
- $r$ for interest rate
- $\sigma$ for standard deviation of stock return (more about this later)

Option buyers and option writers

- Options can be bought or sold
- Like long or short positions in stock or other assets
- **Call option buyer**
  - Pays money up front, gets money if call ends in the money, i.e. if $S_T - X > 0$
- **Call option writer/seller**
  - Gets money up front, pays money if call ends in the money, i.e. if $S_T - X > 0$
Why buy a call on a stock?

- Call right to buy the stock in future
- Spend $0.67 today, if the stock price goes up, you can still buy PG for $79 in 18 days.
- Today, PG is 78.53. Costs me $3 for the right to buy one share of PG on or before 17 Jan 2015 for $80.
  - Why? For $3 you get to bet (apostar) that the stock will go over $80 by 17 Jan 2015
  - You only make money if the stock actually goes over $83
- Another why? Instead of buying the stock today, you get to wait 9 months to buy the stock? DELAYED PURCHASE

Question

- What's worth more?
  - Right to buy PG in next 18 days for $80?
  - Right to buy PG in next 9 months for $80?
- Calls with longer expiration times are worth MORE
- PG January 2015 call with eXercise = $80
  - Costs $3 today
  - Between now and January 2015: Only has non-negative cash flow

PG X=$80 call, T = expiration = 17jan2015, C=cost = $3

- What happens if on 17jan2015, PG stock is worth $100?
  - You exercise the call option: Buy the stock for $80
  - Your immediate profit is $20 ($100 – $80)
  - Taking into account the price you paid, your net profit = $17
- What happens if on 17jan2015, PG stock is worth $150?
  - Your net profit = $67 = $70 - $3 = max[$S_T - X,0] - C
- What happens if on 17jan2015, PG stock is worth $60?
  - Will you use your right to buy the stock for $80? NO!
  - Immediate profit = max[ST - X,0] = 0
  - Net profit: max[ST - X,0] – C = -3

Puts on PG

- Put is the right to SELL a share of PG stock on or before 17Jan2015
- A PG X=$80 January2015 Put, selling today for $6.65
- If next January, PG has stock price of $100, will you want to exercise the put? MEANS: do you want to sell a share of PG for $80, if the stock price is $100? NO
  - Immediate profit in January from the Put? 0
  - Net profit: -6.65
- If, next January, PG has stock price of $50. Will you exercise the put? YES
  - Immediate profit = 30 = 80 – 50 = X – ST
  - Net profit: 30 – 6.65 = 23.35

Profit formula for calls and puts

- Profit on a call = max[ST - X,0] – C
- Profit on a put = max[X-ST,0] - P
Jaime asks

- January X=80 CALL has price today of $3
- January X=80 PUT has price today of $6.65
- Why?
  - Call is a bet that PG will go above $80 by January 2015
  - Put is a bet that PG will go below $80 by January 2015
- Why is the put more valuable than the call? Market thinks that chance of PG going above $80 less than the chance of going below $80.

Diana was asked

- Why does the X=35 PG January 2015 call have a higher price than the X=50 PG January call?
  - X=35 call gives you the right to buy the stock in January for 35
  - X=50 ______ right to buy stock in January for 50
  - Better to be able to buy the stock for 35 than to buy for 50
- Conclusion: As X ↑ call price ↓
- Conclusion: As X ↑ put price ↑

How to think about options

- Call is a bet that the stock price will go up in the future
- Call is a way to delay the purchase of stock
  - Instead of buying stock, buy the right to buy the stock in the future (the call)
- Put is a bet that the stock price will go down in the future
- Put is a way to delay the sale of stock
  - Instead of selling the stock, buy the right to sell the stock in the future (the put)

Option is a one-sided bet

- When you buy a call or put, you can
  - Make an unlimited amount of money in the future
  - You can only lose your original investment
- Finance is full of two-sided bets
  - Stocks: If price goes up/down by $10, you gain/lose $10
  - Futures contract: Contract to buy commodity in future for a given price. Losses/gains are symmetric

Really general property of calls

- The higher the exercise price, the lower the price of the call
  - Call with X = 40 is a bet that the stock price will be > 40 in the future
  - Call with X = 50 is a bet that the stock price will be > 50 in the future (less likely, hence less valuable)
Really general property of puts

- The higher the exercise price, the higher the price of the put
  - Put with $X = 40$ is a bet that the stock price will be < 40 in the future
  - Put with $X = 50$ is a bet that the stock price will be < 50 in the future (more likely, hence more valuable)

Really general property of calls and puts

- Longer-maturity puts and calls are more valuable
  - Call with $X = 40$ and maturity $T = 6$ months
  - Call with $X = 40$ and maturity $T = 1$ year
- Interpretation
  - More time to see stock price rise > 40
  - More time to delay purchase of stock

A more difficult property

- Options are only interesting when the underlying stock is risky.
  - Why place a bet on a non-risky asset?
  - If asset is not risky, everything is known
- The more risky the underlying asset, the more interesting the option.
- Because an option is a one-sided bet (you can’t lose more than you paid):

  The more risky the underlying stock, the more the option is worth.

Yahoo option information

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<th>Stock</th>
<th>Strike</th>
<th>Type</th>
<th>Expiration</th>
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<td>ABC</td>
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<td>Put</td>
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MUST ACTIVE OPTIONS, 22 OCTOBER 2012

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<th>Symbol</th>
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<th>Option Type</th>
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<td>17-Nov-12</td>
<td>150</td>
<td>Put</td>
<td>2000</td>
</tr>
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</table>

In the money is white, out of money in yellow.
Importing data from Web into Excel

- **Data | From Web**
- Opens Microsoft Explorer: Got to URL
- Mark arrows of desired tables

Option profit patterns

- Popular sport! Graph payoffs for call option, put option, stock at exercise date $T$ as function of stock price $S_T$
- Graph payoffs of combinations
  - Two calls with different exercise prices ("spread")
  - Three calls or puts with different exercise prices ("butterfly")

Stock profit pattern

- Make money on bought stock if price rises
- Make money on shorted stock if price falls

Call option profit pattern

- Call buyer’s profit $= \max(0, X - S_T - C)$
- $= \max(0, -50, 0) - 15$
- $= -15$ if $S_T \leq 50$
- $= (X - 50) - 15$ if $S_T > 50$

Put option profit pattern

- Put buyer’s profit $= \max(0, X - S_T + P)$
- $= \max(0, 50 - S_T - 10)$
- $= -10$ if $S_T \leq 50$
- $= 40 - S_T$ if $S_T > 50$
Protective put

- Buy stock today at price $S_0$
- But a put for $P_0$ with exercise price $X$
- Cost today:
  - $S_0 + P_0$
- Payoff at time $T$:
  - $S_T + \max(X - S_T, 0)$

Protective put payoff pattern looks like that of a call. Is it true that:
- $S_T + P_T = \text{Call}$
- Almost but not quite—see "Put-Call Parity" (later on)

Call spread

- Buy one call with exercise price $X_{\text{high}}$
- Write call with exercise price $X_{\text{low}}$
- Both with same time $T$ to maturity
- Profit at $T$:
  - $-\max(S_T - X_{\text{high}}, 0) + \max(S_T - X_{\text{low}}, 0)$

Butterfly

- Combination of three puts or calls with different exercise prices (some long, some short)
- Total number of positions (short + long) adds to zero
**Option arbitrage propositions**

- Facts about option prices
- Derived without much/any assumptions about stochastic process of stock price
- Derived only from definitions

**Arbitrage position 0**

- Consider an American call costing $C_0$, with exercise price $X$, where the stock price is $S_0$. Then
  
  \[ C_0 \text{ must be } > \text{Max}(S_0 - X, 0) \]

- Proof by example: Suppose $C_0 = 5$, $S_0 = 50$, and $X = 40$.

- **Make immediate profit**
  
  - Buy call: \(-5\)
  - Exercise immediately: \(-40\)
  - Sell stock immediately: \(+50\)

**Arbitrage proposition 1**

- Consider a European call costing $C_p$ with exercise price $X$, where the stock price is $S_p$. Then
  \[ C_p \text{ must be } > \text{Max}(S_p - PV(X), 0) \]

- Proposition 0 is trivial

**Arbitrage proposition 2**

- It is never optimal to early-exercise an American call written on a stock which doesn’t pay dividends before the option maturity $T$.

- Another interpretation: If you’re thinking about early-exercising a call:
  
  - SELL THE CALL, don’t exercise it
  - You’ll make more money
Proof by example of Prop. 2

- You own a call with 0.5 years to maturity.
  - Call exercise, $X=50$
  - Current stock price, $S=80$
  - Interest rate, $r = 6\%$

- Immediate early exercise:
  
  \[ \text{Payoff} = S - X = 30 \]

- By Prop. 1, market price of call is at least
  \[ \max(S - PV(X), 0) = \max(80 - (\exp^{-0.5 \times 6\%})50, 0) = 31.45 \]

- Better off selling the call than exercising

Conclusion from Prop. 2

- The American feature of calls is often worthless
- In many cases: American call and European call have same value
- Not true for puts: European put worth less than American put

Proposition 3: Put-call parity

- Consider a European put and call on the same stock. Put and call have same exercise price $X$. Stock pays no dividends before option exercise date $T$.

Then:

\[ P_0 + S_0 = C_0 + PV(X) \]

where

- $P_0$ is the price of put plus stock
- $S_0$ is the price of call plus present value of exercise

Proposition 5: Call price convexity

- Consider three calls on same stock with same maturity $T$. Assume that Call1 has exercise price $X_1$, Call2 has $X_2$, Call3 has $X_3$.

- Assume $X_1 < X_2 < X_3$ and equally spaced: $X_2 = (X_1 + X_3)/2$.

Then

\[ Call2 < \frac{Call1 + Call3}{2} \]

Proposition 5 example and counterexample

- Use butterfly spread previously illustrated
- When Proposition 5 condition is violated, there is an arbitrage opportunity.
- Violation of Prop. 5:

\[ Call2 > \frac{Call1 + Call3}{2} \]

No arbitrage:

\[ Call2 < \frac{Call1 + Call3}{2} \]

Sometimes you win, sometimes you lose.
Arbitrage: \( \text{Call2} \geq \frac{\text{Call1} + \text{Call3}}{2} \)

<table>
<thead>
<tr>
<th>Exercise Price</th>
<th>Call1</th>
<th>Call2</th>
<th>Call3</th>
</tr>
</thead>
<tbody>
<tr>
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<td>10</td>
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</tr>
<tr>
<td>30</td>
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</tr>
<tr>
<td>40</td>
<td>4</td>
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**PROPOSITION 5 CONDITION VIOLATED**