A Stochastic Movement Simulator

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This talk will cover the following.

1. Define a stochastic movement simulator
2. Estimate its parameters
3. Tips for working with wildlife agencies

Overview of animal movement modeling

One of the earliest animal movement models was a 1994 paper on bison foraging behavior bison in Yellowstone.

Many such models can be categorized as mainly concerned with modeling animals' *habitat selection* (steady state movement dynamics) or a one-time movement across the landscape known as *dispersal*.

Increasingly relevant

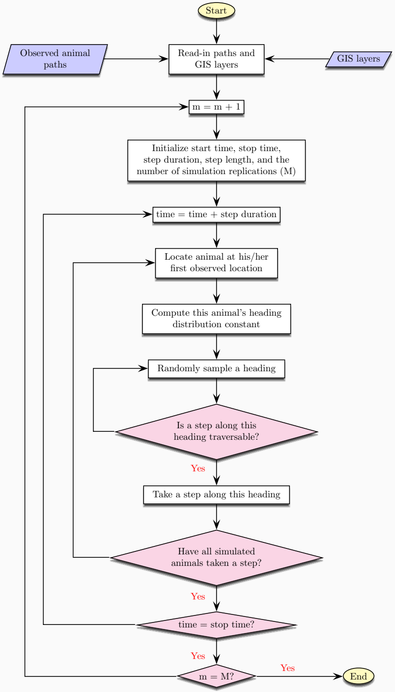
Movement ecology is becoming important to wildlife management, see Allen and Singh (2016). "Linking movement ecology with wildlife management and conservation," *Frontiers in Ecology and Evolution*.

Two *Movement Ecology* papers

1. Buchin et al. (2015), "Deriving movement properties and the effect of the environment from the Brownian bridge movement model in monkeys and birds."
2. Bohrer et al. (2014), "Elephant movement closely tracks precipitation driven vegetation dynamics in a Kenyan forest-savanna landscape."

Grids or continuous space?

1. Models for which the landscape is partitioned into a grid of cells are easy to program and easy to fit to available computational resources through adjustments to the model's cell size.
2. But continuous-space models are realistic and can be directly related to GIS input and output once coordinate systems are aligned.


Start
Observed animal paths
Read-in paths and GIS layers
GIS layers
m = m + 1
Initialize start time, stop time, step duration, step length, and the number of simulation replications (M)
time = time + step duration
Locate animal at his/her first observed location
Compute this animal's heading distribution constant
Randomly sample a heading
Is a step along this heading traversable?
Yes
Take a step along this heading
Have all simulated animals taken a step?
Yes
time = stop time?
Yes
m = M?
Yes
End

Random heading

Say that an animal moves a step of size s along a heading that the animal thinks will maximize its utility. There is some noise in the animal's decision so that the heading ultimately chosen is a stochastic function of utility.

This stochasticity is modeled by first computing a utility function in terms of h; denoted $u(h)$, and then tabulating a probability distribution that is proportional to this utility.

One step

A deviate is drawn from this heading distribution by evaluating the associated quantile function at a value that is randomly drawn from the unit-interval uniform distribution. Finally, the animal moves one step along this randomly-drawn heading.

Let $\mathbf{x}$ be the vector containing all variables that influence the animal's utility function. Let $\underline{\beta}$ contain the associated parameters. Say that animal i is presently located at $\mathbf{l}_0$.

What rhinos like

For instance, rhinos like to eat and mate. Let $twi(\mathbf{l}_0, t)$ be the *topographic wetness index* value at location $\mathbf{l}_0$ and time t.

Let $d_f(\mathbf{l}_0, h, t)$ be the distance to the closest food from location $\mathbf{l}_0$ along heading h at time of year t.

Demographics and mate locations

Likewise, let $d_m(\mathbf{l}_0, h, t)$ be the closest potential mate from location $\mathbf{l}_0$ along heading h at time of year t. Let $\mathbf{c}_i$ be the demographics of rhino i, e.g. gender, and age.

Then $\mathbf{x}' = (twi(\mathbf{l}_0, t),\ d_f(\mathbf{l}_0, h, t),\ d_m(\mathbf{l}_0, h, t),\ \mathbf{c}_i')$ and $u(h) = \mathbf{x}'\underline{\beta}$.

Heading distribution

The animal will take a step of size s in the direction h.

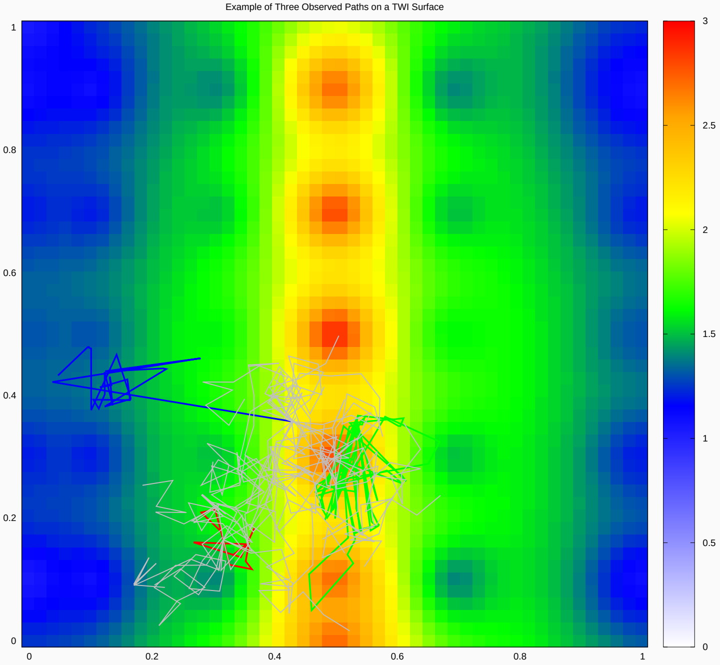
We assume that the animal has drawn (in-effect) the value h from a distribution defined by the probability density function (PDF) $f(h) = u(h) / \int_0^{360} u(y)dy$. Call this the *heading distribution*.

A heading is repeatedly drawn until the path defined by that heading and the pre-determined step size is traversable by the animal.

SMS algorithm

First, n animals are assigned starting locations. Then, at time t, each animal takes a step of size s along a heading, h_q found by randomly drawing from the animal's heading distribution.

This random draw is accomplished by finding the root of the function $g(h) = p - F(h)$ where p is the deviate drawn from the uniform distribution over the unit interval, and $F(h)$ is the Cumulative Distribution Function (CDF).



Example of Three Observed Paths on a TWI Surface
0
0.2
0.4
0.6
0.8
1
0
0.2
0.4
0.6
0.8
1
0
0.5
1
1.5
2
2.5
3

Numerical integration

This CDF is formed by numerically integrating the heading distribution's PDF over the interval $(0,\ h)$.

An adaptive Newton-Cotes nine-point scheme is used to perform the numerical integration, and Brent's method is used to find the root of $g(.)$.

Each member of the simulated animal population continues to take such steps until the simulation's end-time is reached.

Westing correction

Because the mean of this uniform distribution is π, a bias towards a westing heading is possible.

This potential bias is avoided by rotating the heading distribution coordinate system through a random angle each time a step is simulated.

Statistical estimation of SMS parameters

Classical maximum likelihood needs a mathematical form of the likelihood of each and every possible sample.

Simulation models rarely possess such a function.

If the simulation model is *simulable*, the unknown likelihood function may be approximately maximized with Maximum Simulated Likelihood Estimation (MSLE) as it replaces the actual likelihood with a simulated approximation to it.

SMS parameter estimation

Say that n animals have been observed moving through the landscape over a time interval.

Let p_j be the path of the j^{th} observed animal. A *path* is an ordered list of vertices, $P = \{v_1, \ldots, v_n\}$.

Let r_{ijk} be the dissimilarity between the path taken by the i^{th} simulated animal during the k^{th} simulation run and the j^{th} observed animal that shares the same demographic values.

Path dissimilarity

Our measure of path dissimilarity, due to Martí et al. (2009), is:

$$r = \frac{1}{2}\left[\frac{\sum_{v_i \in P_1} \delta(v_i, P_2)}{|P_1|} + \frac{\sum_{u_j \in P_2} \delta(u_j, P_1)}{|P_2|}\right]$$

where $\delta(v, P_1) = \min_{v_j \in P_1} \delta(v, v_j)$ and $|P_i|$ is the number of vertices in path P_i.

Nonparametric estimation of path PDFs

Say that animal movement over the observed time interval has been simulated m times.

The path's PDF at the j^{th} observed path, p_j may be approximated with a v nearest-neighbor, nonparametric density estimator by setting it equal to the inverse of the (scaled) dissimilarity between p_j and the v^{th} most-similar simulated path.

PDF estimation, continued

Let $\tilde{f}(p_j)$ denote this approximate PDF. The value of v equals $\alpha \times m$ where α is a small value between 0 and 1 (typically about 0.05).

The idea here is that the PDF at path p_j becomes small as the dissimilarity between p_j and the v^{th} most-similar simulated path increases. Denote the distance between two points in the landscape with $\delta(u, v)$.

Log-likelihood approximation

The log-likelihood is approximated with $\sum_{j=1}^{n} \log \tilde{f}(p_j)$. Parameters are adjusted such that this approximate log-likelihood is maximized.

Movement behavior indices

A multivariate approach to goodness-of-fit is to define several movement behavior indices, model them as a multivariate random vector, and create diagnostic plots to assess model-data agreement.

Straightness

To this end, quantify a path's *tortuosity* with its *straightness*, and *sinuosity*.

$ST = dE/L$ where dE is the Euclidean distance between the beginning and end of the path; and L is the path's length.

Sinuosity

$$SI = 2\sqrt{p\left\{\frac{1-c^2-s^2}{(1-c)^2+s^2}+b^2\right\}}$$

where p is the mean step length, c is the mean cosine of the path's turning angles, s is the mean sine of the path's turning angles, and b is the coefficient of variation of step length.

Search effort and migration indices

A path's spatial range, R (the path's overall spatial range) is a measure of the animal's search effort.

Let *net displacement* be the distance between the path's starting location and a subsequent path location.

MND and *SND*

MND: average net displacement

SND: standard deviation of net displacement

A resident has low values of both *MND* and *SND*. A transient has a high *MND* value and a low *SND* value.

Heatmap

Realizations of a multivariate random vector may be portrayed with a *heatmap*.

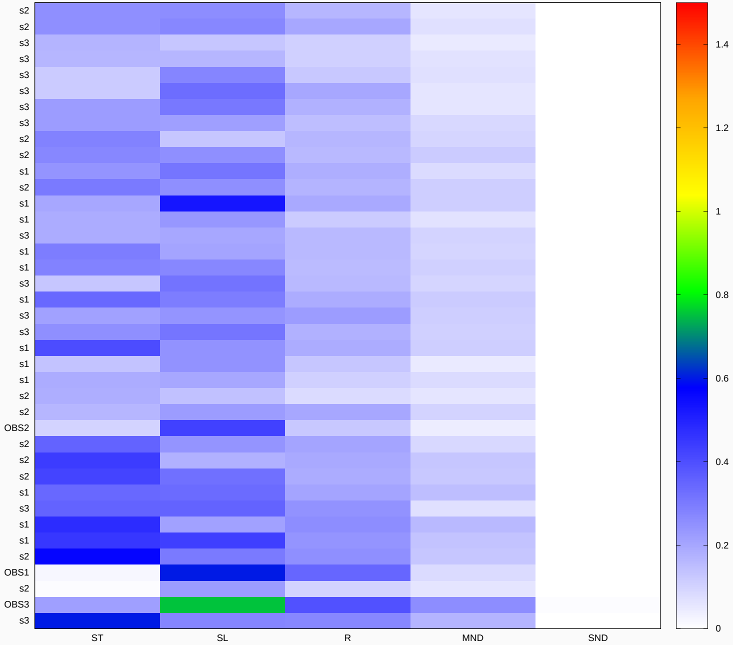
Each path is assigned a row, and each variable, a column in a matrix. The value of a path variable is indicated by a color in the associated box.

A path's row is determined by its density value computed under the fitted SMS model.

The following is a heatmap of the movement behavior indices (ST, SI, R, MND, SND) of the observed paths along with those simulated with the fitted model.

The path label "OBSi" indicates the path of the i^{th} observed rhino, and "si" indicates a simulated path of this animal.

Goodness of fit improves as the observed path rows are more evenly mixed-in with those of simulated paths.


s2
s2
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s1
s3
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s3
s3
s1
s1
s1
s2
s2
OBS2
s2
s2
s2
s1
s3
s1
s1
s2
OBS1
s2
OBS3
s3
ST
SL
R
MND
SND
1.4
1.2
1
0.8
0.6
0.4
0.2
0

Working with a wildlife agency

1. Do some free work for them and then have a chat.
2. Propose a formal but unfunded project.
3. Now, jointly approach a conservation organization for funding.
4. Be patient but persistent about acquiring data.
5. Move with them.